2011 International Conference on Signal, Image Processing and Applications
With workshop of ICEEA 2011
IPCSIT vol.21 (2011) © (2011) IACSIT Press, Singapore

Robust Model Reference Adaptive Control of Linear Multivariable Plants- a New Approach

Adhish Kr. Chakrabarty[1+] and Samar Bhattacharya[2]
[1] Dept. of App. Electronics & Instrumentation Engg, Guru Nanak Inst. of Technology, Kolkata-114, India.
[2] Electrical Engineering Department, Jadavpur University, Kolkata-32, India.

Abstract. A direct robust model reference adaptive control for linear plants with time-varying structured uncertainties is presented in this paper. The proposed robust MRAC uses the estimates of the structured uncertainty to generate the control input. The output error equation is used to estimate the uncertainty. The control signal is applied to the plant simultaneously with the reference input, resulting in nearly zero tracking error. It is also shown that the proposed method does not require any knowledge about the bound of the uncertainty to achieve tracking, which is required by many other methods. Lyapunov's theory is used to achieve stability of the plant and boundedness of signals. Simulations are used to show the effectiveness of the proposed method.

Keywords: model reference adaptive control, uncertainty estimation, Lyapunov's theory

1. Introduction

The first model reference adaptive control method and other early MRAC methods dealt with time-invariant parameters ([1], [2]). Practical situations like aircraft flight control, process control, robotics etc., where time-varying parameters are frequently come across, led to the development of robust MRACs. These early schemes also ensured only bounded tracking error, some of which e.g. σ-modification, switching σ-modification, ε_1-modification etc. are discussed in [3]. Some other robust MRACs were developed for special types of parametric uncertainties i.e. finite jump parameters [4], periodical parameter variation [5] and parameter variations exponentially decaying to zero [6].

Another technique to develop robust MRAC involves using robust control laws to achieve robustness and then using MRAC schemes to make actual plant output track model plant output [7], [8].

In [9] MRAC is developed by estimating what the control signal would be if the plant parameters were known. In [10] a state-feedback MRAC is developed for systems with bounded arbitrary parameter variations, which is an improvement over the method discussed in [11].

However, the method proposed in [10] suffers from the drawback that it considers only the uncertainties associated with system matrix. The input matrix is considered to be the same for both the actual plant and model plant, though it can not be guaranteed that the input matrices of both the actual and model plants will be same.

The present work provides a method that overcomes the drawback of the method given in [10] by comnsidering plants with uncertainties both in the system matrix and the input matrix. This method basically extends the easy to use techniques to develop MRAC for linear time-invariant plants as given in [3], to time varying plants. In the present method the actual plant is subjected to simultaneous actions of the reference input and a control input. The control input is generated with the help of the estimates of the uncertainties

[1 +] Corresponding author. Tel.: + 919830723659; fax: 91-33-25637957
E-mail address: cadhish@yahoo.co.in.

associated with system matrix and input matrix. It is assumed that the certainty equivalence principle holds. Lyapunov's method is used to obtain bounded control input.

It is shown that the method proposed here does not require any knowledge about the nature of the uncertainties associated with the actual plant and it provides good tracking.

2. Problem Statement

The linear time-invariant model plant is represented by

$$\dot{x}_m = A_m x_m + B_m r \tag{1}$$

$y_m = Cx_m$, where $x_m \in \mathbb{R}^{n\times 1}$ is the model plant state, $y_m \in \mathbb{R}^{m\times 1}$ is the actual plant output, $r \in \mathbb{R}^{p\times 1}$ is the reference input and matrices $A_m \in \mathbb{R}^{n\times n}$ and $\eta \in \mathbb{R}^{n\times p}$ are the model state and input matrices, respectively. Now, consider a multivariable linear time-varying actual plant

$$\dot{x}_p = A_p(t)x_p + B_p u(t) \tag{2}$$

$y_p = Cx_p$, where $x_p \in \mathbb{R}^{n\times 1}$ is the actual plant state, $y_p \in \mathbb{R}^{m\times 1}$ is the actual plant output, $u(t) \in \mathbb{R}^{p\times 1}$ is the input (generally, the control input) and matrices $A_p \in \mathbb{R}^{n\times n}$, $B_p \in \mathbb{R}^{n\times p}$ and $C \in \mathbb{R}^{m\times n}$ are the state matrix, input matrix and output matrix, respectively. Now, the actual plant equation is rewritten as

$$\dot{x}_p = (A_m + \delta(t))x_p + (B_m + \eta)u(t) \tag{3}$$

$y_p = Cx_p$, where $\delta(t) \in \mathbb{R}^{n\times n}$ the time-varying is structured uncertainty affecting the plant states and $\eta \in \mathbb{R}^{n\times p}$ is the uncertainty associated with input matrix. If x_p can be made to track x_m, y_p will track y_m. To obtain the state-tracking, reference input r and a separate control signal s are applied to the actual plant. Therefore equation (3) is remodeled as

$$\dot{x}_p = (A_m + \delta(t))x_p + B_p(r + ks) \tag{4}$$

Where, $s \in \mathbb{R}^{p\times 1}$ is the control signal and k is constant to be chosen by the designer.

The method given in [10] uses the plant equation:

$$\dot{x} = (A + \Delta A(t))x_p + B\,u(t) \;, x(0) = x_0,$$

$y = Cx$ and the control signal given by [10] is

$u(t) = (B^T B)^{-1} B^T (A_m - A(t))x(t) - (B^T B)^{-1} B^T v(t) + r(t)$, where $v(t)$ is the feedback term in the uncertainty estimation algorithm, representing an adaptive estimate of $\Delta A(t)x(t)$ [10].

3. Proposed MRAC

Let the state-tracking error be given by e_o.

$e_o = x_p - x_m$. To make the tracking error zero, from equations (1) and (4), the control signal is:

$B_p ks = -\delta(t)x_p - \eta r$, which leads to zero tracking error when $A_p - \delta(t) = A_m, B_p - \eta = B_m$.

Now, let $B_p = BK_1$ where $K_1 \in R^{n\times m}$ a time-invariant constant matrix is and $B \in R^{n\times n}$, also a constant matrix, is called the state-interaction matrix and it is so chosen that it is invertible.

$$\therefore BK_1 ks = (-\delta(t)x_p - \eta r)$$

$$\Rightarrow s = k^{-1}(K_1^T K_1)^{-1} K_1^T B^{-1}(-\delta(t)x_p - \eta r)$$

Of course, K_1 is so chosen that $K_1^T K_1$ is invertible. Now, equation (4) can be rewritten as

$$\dot{x}_p = A_m x_p + \delta(t)x_p + B_m r + \eta r + B_p ks \tag{5}$$

$$\Rightarrow \dot{x}_p = A_m x_p + B_m r + (\delta(t)x_p + \eta r + B_p ks)$$

$$\Rightarrow x_p(D - A_m) = B_m r + (\delta(t)x_p + \eta r + B_p ks)$$

$$\Rightarrow x_p = x_m + (\delta(t)x_p + \eta r + B_p ks)(D - A_m)^{-1}$$

$$\Rightarrow e_o = x_p - x_m = (\delta(t)x_p + \eta r + B_p ks)(D - A_m)^{-1} \tag{6}$$

The estimate of the tracking error can be generated by:

$$\hat{e}_o = (\delta(t)x_p + \eta r + B_p k\hat{s})(D - A_m)^{-1}$$
$$= (\delta(t)x_p + \eta r - \hat{\delta}(t)x_p - \hat{\eta}r)(D - A_m)^{-1}$$

Now, assuming the certainty equivalence principle [1] to hold, $\delta(t) = \hat{\delta}(t)$ and $\eta = \hat{\eta}$ are obtained $\therefore \hat{e}_o = (0)(D - A_m)^{-1} = 0$. The estimation error is given by:

$$\varepsilon = e_o - \hat{e}_o$$
$$\Rightarrow \varepsilon = e_o$$

4. Parameter Estimation

Output error equation is used to estimate the uncertainty as it gives better performance in a noisy environment [3]. Equation (6) is rewritten as

$$e_o = (\tilde{\delta}(t)x_p + \tilde{\eta}r)(D - A_m)^{-1}, \text{ where } \tilde{\delta}(t) = \delta(t) - \hat{\delta}(t) \text{ and } \tilde{\eta} = \eta - \hat{\eta}$$
$$\Rightarrow \dot{e}_o = A_m e_o + \tilde{\delta}(t)x_p + \tilde{\eta}r$$

Following the method given in [3], let the following function be a candidate for Lyapunov's function:

$$V(e_o, \delta_A, \delta_B) = e_o^T P e_o + tr\left(\frac{\tilde{\delta}_A{}^T P \tilde{\delta}_A}{\gamma}\right) + tr\left(\frac{\tilde{\delta}_B{}^T P \tilde{\delta}_B}{\lambda}\right) \qquad \text{(sec.6.2.3 [3])}$$

where, $tr(A)$ denotes the trace of matrix A. $\gamma, \lambda > 0$ are constant scalars.

$P = P^T > 0$ is chosen as the solution of the Lyapunov equation $A_m^T P + PA_m = -Q$, whose existence is guaranteed by the stability of A_m (theorem 3.4.10,[3]). Now, taking derivative of V w.r.t. time we get

$$\dot{V} = \dot{e}_o^T P e_o + e_o^T P \dot{e}_o + tr\left(2\frac{\tilde{\delta}_A{}^T P \dot{\tilde{\delta}}_A}{\gamma}\right) + tr\left(2\frac{\tilde{\delta}_B{}^T P \dot{\tilde{\delta}}_B}{\lambda}\right)$$

$$\left(\because \frac{\dot{\tilde{\delta}}_A{}^T P \tilde{\delta}_A}{\gamma} = \frac{\tilde{\delta}_A{}^T P \dot{\tilde{\delta}}_A}{\gamma} \text{ and } \frac{\dot{\tilde{\delta}}_B{}^T P \tilde{\delta}_B}{\lambda} = \frac{\tilde{\delta}_B{}^T P \dot{\tilde{\delta}}_B}{\lambda}\right) \quad \text{(sec.6.2.3 [3])}$$

Now, $\dot{V}$ along the tracking error trajectory is:

$$\dot{V}\big|_{\dot{e}_o} = e_o^T (A_m^T P + PA_m) e_o + x_p^T \tilde{\delta}_A{}^T P e_o + r^T \tilde{\delta}_B{}^T P e_o + e_o^T P \tilde{\delta}_A x_p + e_o^T P \tilde{\delta}_B r$$
$$+ tr\left(2\frac{\tilde{\delta}_A{}^T P \dot{\tilde{\delta}}_A}{\gamma}\right) + tr\left(2\frac{\tilde{\delta}_B{}^T P \dot{\tilde{\delta}}_B}{\lambda}\right) \qquad (\because P = P^T)$$

$$\Rightarrow \dot{V}\big|_{\dot{e}_o} = -e_o^T Q e_o + 2x_p^T \tilde{\delta}_A{}^T P e_o + 2r^T \tilde{\delta}_B{}^T P e_o + tr\left(2\frac{\tilde{\delta}_A{}^T P \dot{\tilde{\delta}}_A}{\gamma}\right) + tr\left(2\frac{\tilde{\delta}_B{}^T P \dot{\tilde{\delta}}_B}{\lambda}\right)$$

$$\left(\because e_o^T P \tilde{\delta}_A x_p \in \mathbb{R}^{1\times 1} \therefore e_o^T P \tilde{\delta}_A x_p = (e_o^T P \tilde{\delta}_A x_p)^T = x_p^T \tilde{\delta}_A{}^T P e_o \text{ .Similarly, } e_o^T P \tilde{\delta}_B r = r^T \tilde{\delta}_B{}^T P e_o\right)$$

$$\Rightarrow \dot{V}\big|_{\dot{e}_o} = -e_o^T Q e + 2tr\left(\frac{\tilde{\delta}_A{}^T P \dot{\tilde{\delta}}_A}{\gamma} + \tilde{\delta}_A{}^T P e_o x_p^T + \frac{\tilde{\delta}_B{}^T P \dot{\tilde{\delta}}_B}{\lambda} + \tilde{\delta}_B{}^T P e_o r^T\right)$$

Now, $\dot{V}\big|_{\dot{e}_o}$ will be negative if

$$\frac{\tilde{\delta}_A^T P \dot{\tilde{\delta}}_A}{\gamma} = -\tilde{\delta}_A^T P e_o x_p^T$$
$$\Rightarrow \dot{\tilde{\delta}}_A = -\gamma e_o x_p^T$$

$$\Rightarrow \dot{\hat{\delta}}_A = \gamma e_o x_p^T \text{. Similarly, } \dot{\hat{\delta}}_B = \lambda e_o r^T$$

$\therefore V$ is a Lyapunov function for the plant under consideration and the plant is universally stable. It can be shown (3) that $\dot{e}_o \in L_\infty$ and $e_o \in L_2$, which imply that $e_o \to 0$ as $t \to \infty$.

5. Simulation Results

The simulations are carried out for two cases. In case-I, only the state matrix parameters are affected by uncertainty and input matrix parameters are same for both the model plant and actual plant. In case-II, the system matrices are affected by sinusoidal uncertainty and model plant input matrix and actual plant input matrix are different.

Model plant state matrix and model plant input matrix are:

$A_m = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix}$ and $B_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, respectively. $BK_1 = \begin{bmatrix} -2 & -1 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $K_1^T K_1 = 5$ and $k = 50$.

Case-I.a: sinusoidal uncertainty

The uncertainty affecting the actual plant system parameters are given by: $\delta(t) = 0.1\sin 10t$

Case-I.b: random uncertainty:

The uncertainty is given by a random signal and its magnitude is within ±0.1

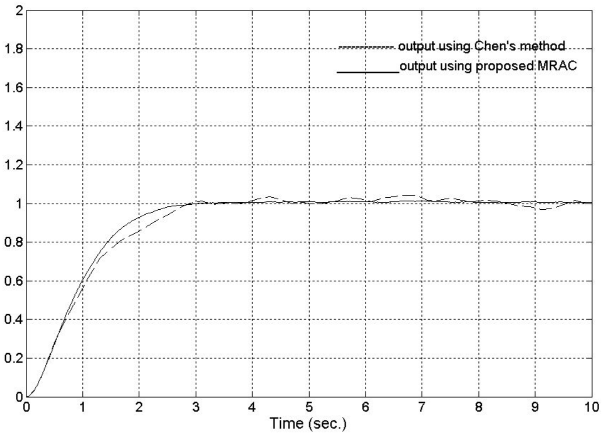

Fig.1.Actual plant output (case-I.a)

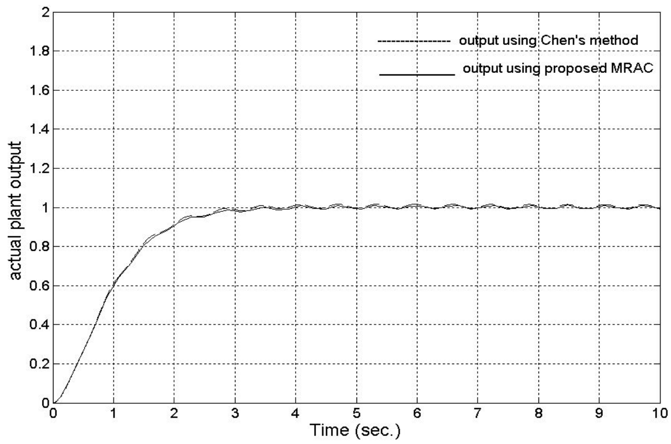

Fig.2.Actual plant output (case-I.b)

Case-II: input matrices are different: In this case, the uncertainty affecting the system parameters is given by: $\delta(t) = 0.1\sin 10t$ and the actual plant input matrix is given by: $B_p = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

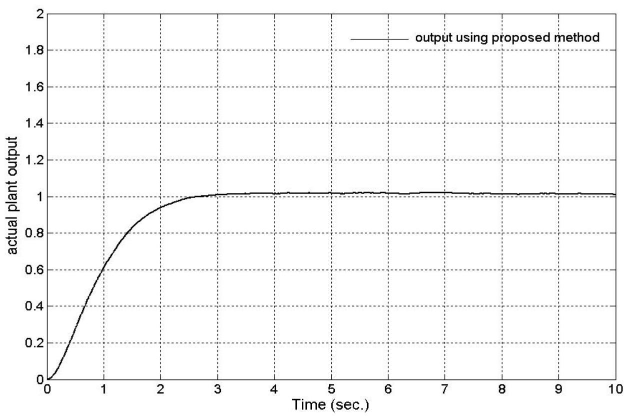

Fig.3. Actual plant output (proposed MRAC)

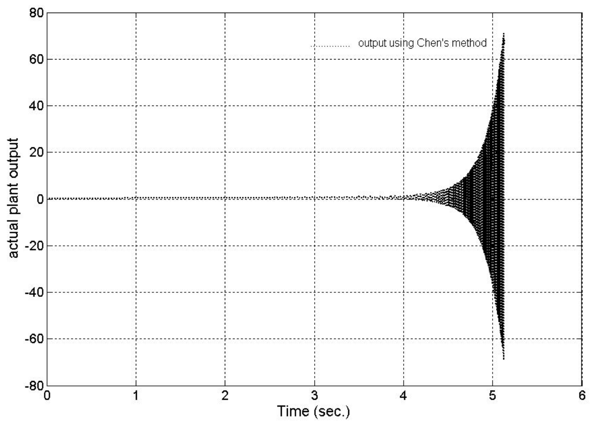

Fig.4. Actual plant output (MRAC in [10])

6. Conclusion

The robust MRAC proposed in this work is shown to work satisfactorily in the presence of different bounded structured uncertainties. The present method is easier to implement and analyze and more general in scope than the method given in [10]. There can be many solutions that satisfy the relation $B_p = BK_1$, but the differences are only in the rate of adaptation, which can be taken care of by setting the adaptive gain k properly.

7. References

[1] S.Shastry and M.Bodson, *Adaptive Control- Stability, Convergence and Robustness*, Prentice-Hall, Englewood Cliffs, NJ, 1989.

[2] K.J.Astrom and B.Wittenmark, *Adaptive Control*, Pearson Education, Inc. 1995.

[3] P.Ioannou and J.Sun, *Robust Adaptive Control*, Prentice Hall, Inc. 1996.

[4] JM. Martin-Sanchez, "Adaptive Control for Time Variant Processes", *International Journal of Control*, vol. 44, pp. 315-329, 1986.

[5] F. Ohkawa, "Model Reference Adaptive Control Systems for Linear Time-Varying Systems with Periodically varying Parameters and Time-delays", *International Journal of Control*, vpl.44, pp. 171-179, 1986.

[6] G.C. Goodwin, D.J.Hill, X. Sianya, "Stochastic Adaptive Control for Exponentially Convergent Time-varying Systems", *Proc. Of 23rd Conference on Decision and Control*, vol. 39, 1984.

[7] P.O.Gutman, "Robust Certainty Equivalence- A New Principle for Adaptive Control", *Proc. of IFAC Workshop on Robust Adaptive Control*, Newcastle, Australia, pp. 113-118, 1988.

[8] P.O. Gutman, "Robust and Adaptive control: Fidelity or an Open Relation?" *Systems and Control Letters,* vol. 49, issue 1, pp. 9-19, 2003.

[9] D.E.Miller, "A New Approach to Model Reference Adaptive Control", *IEEE Transactions on Automatic Control*, vol.48, no.5, pp. 743-757, 2003.

[10] Min-Shin Chen, Jia-Ming Wu, "A New Model Reference Adaptive Control for Linear Time-varying Systems", *International Journal of Adaptive Control and Signal Processing,*, vol. 14, Issue:4, pp. 469-479, 2000.

[11] A.M. Annaswamy and K.S. Narendra, "Adaptive Control of Simple Time-Varying Systems", *Proc. Of 28th Conference on Decision and Control*, pp. 1014-1018, 1989.