

A New Shape Descriptor Based on the Radon Transform: the $R\theta$ -signature

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Abstract. Object recognition has been a topic of research for decades, it operates by making decisions based on the values of several shape properties measured from an object's image. In this paper, a new exploitation of the Radon Transform (RT) is proposed to extract only one projection according to a single angle. This projection is chosen in way that contains the necessary information to recognize an object (a shape descriptor). This descriptor (called $R\theta$ -signature) provides global information of a binary shape regardless its form. The signature keeps fundamental geometrical transformation like scale, translation and rotation. The experiment results on images base shows the efficient of our descriptor in the distinction between rectangular and no-rectangular forms, better than a previous signature based also on Radon transform called R-signature.

Keywords: shape descriptor, radon transform, $R\theta$ -signature.

1. Introduction

As the holy grail of computer vision research is to tell a story from a single image or a sequence of images, object recognition has been studied for more than four decades [4] [8]. This object identification is made in the forms analysis phase which generally occurs after a step of image segmentation. Once the shapes are extracted from the image, they must be simplified before a matching can be made. The simplified representation of forms is often called the shape descriptor or signature. This is an abstraction of a structured model that captures most of the important information of the form. These simplified representations are easier to handle, store and compare than the forms directly. The descriptors for antithetical shapes should be different enough that the shapes can be discriminated. Therefore instead of directly comparing two objects, their shape descriptors are compared. So instead of directly comparing two models, both models are compared by comparing their shape descriptors.

Although some researches has been done in terms of rectangularity, which can be an advantageous characteristic to extract useful tasks such as filtering of images to find parts of potential road and buildings in a satellite image. There are many attempts to measure the rectangularity. The standard method, the Minimum Bounding Rectangle method (MBR), responds unequally to protrusions and indentations, and is sensitive to noise (especially protrusions). Moreover, in [6] the Radon Transform (RT) is used to calculate the R-signature (i.e. the square of the RT) which just characterizes very well the shape of the filled and not emptied object (i.e. Object contour only). In this approach, the R-signature of an object is compared to a theoretic R-signature which represents a perfect rectangle and calculates the similarity between them.

Here in we tried to locate and extract a signature from the Radon space itself without any modification in the RT formula. Our work is motivated by the study introduced by Magli et al. in [3] twelve years ago, where they provide the existence of projection according to an angle from the Radon space, which contains the

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information, can present the object. Actually our work is finding the method to locate this projection which we called the $R\theta$ -signature.

This paper is outlined as follows. After we recall the definition and the properties of the RT in sections 2, we present in Section 3 our $R\theta$ -signature approach while Experiments and Discussion are given in Section 4. Finally, we summarize our research and conclude the paper in Section 5.

2. The Radon Transform

By definition the Radon transform [5], of an image is determined by a set of projections of the image along lines taken at different angles. For discrete binary image data, each non-zero image point is projected into a Radon matrix. Let $f(x, y)$ be an image. Its Radon transform is defined by [1]:

$$R(\rho, \theta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \delta(\rho - x \cos \theta - y \sin \theta) dx dy. \quad (1)$$

Where $\delta(\cdot)$ is the Dirac function, $\theta \in [0, \pi]$ and $\rho \in]-\infty, \infty[$. The Radon transform has several useful properties. Some of them are relevant for shape representation: A translation of f results in the shift of its transform in the variable ρ by a distance equal to the projection of the translation vector on the line $\rho = x \cos \theta + y \sin \theta$. A rotation of the image by an angle θ_0 implies a shift of the RT in the variable θ . A scale of f results the same in both the ρ coordinates and the amplitude of the transform.

To represent the RT of an image, we take multiple, parallel-beam projections of the image from different angles by rotating the source around the centre of the image. The Fig.1 shows a single projection at a specified rotation angle. For example, the line integral of $f(x, y)$ in the vertical direction is the projection of $f(x, y)$ onto the x-axis; the line integral in the horizontal direction is the projection of $f(x, y)$ onto the y-axis [9]. The RT is robust to noise, provided with fast algorithms, and it projects a two-dimensional function into one-dimensional function, for all this reasons, we have decided to employ the RT.

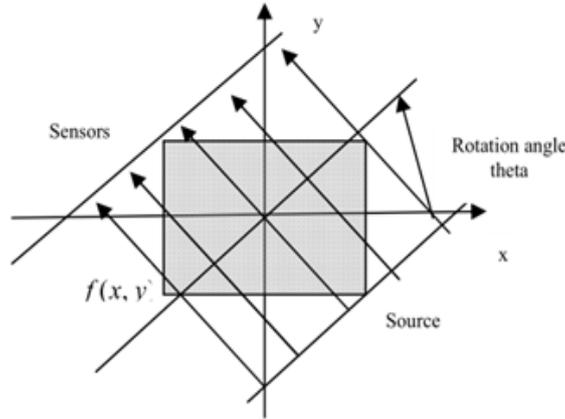


Fig. 1: Parallel-Beam Projection at Rotation Angle Theta.

3. The $R\theta$ -signature

In our study a new exploitation of the RT is proposed. Usually the RT used only in the detection of straight lines. In our work we provide global information of a full binary shape, whatever its form; scale and orientation are, by generating a new signature ($R\theta$ -signature). We define this signature using the RT's property proved by Magli et al. [3], which is the existence of a peak among the Radon projections, which exhibits the same shape of the projected object in the ρ direction.

To find this projection (we called the $R\theta$ -signature), Magli et al. use the wavelet transform. They calculate the similarity between the mother wavelet (look alike the projected object) and the peak in each Radon project. The projection with the highest degree of similarity selected as the represented projection of the object. The disadvantages of this approach are:

- Several mother wavelet functions are used because the shape of the peak is generally unknown
- The setting of these wavelets is not trivial

Here we present an approach that allows the extraction of the projection of a simpler and more efficient way. It can also work with an object of any shape.

3.1. The concept of the R θ -signature

We thought to seek the projection the most revealing form of projected object, i.e., the one that provides the most information about the object in question.

This amount to look for the angle θ which gives the perfect projection, this projection is the "R θ -signature".

In the fact, since the RT is linear by definition, geometric properties like straight lines can be made explicitly by the RT which concentrates energies (loci of intersection of several sinusoidal curves), from the image in few high-valued coefficients in the transformed domain.

Moreover, we can consider an object as a lot of parallel lines put one beside the other, which means that in the right angle projection, those lines will raise a lot of peaks in the Radon space. So if we calculate the sum of the Radon coefficients in this projection, this sum will be higher than the others. Based in this analyse, we sum the Radon values of ρ in each θ angle and we pick the angles with the higher sum. The projections Fig.2 (c) and Fig.2 (d) satisfy this condition.

Then a study on the distribution coefficients of Radon in each selected projection is made. We seek the maximum coefficient in each selected projection and calculate the division ratio of the projection p :

$$DRp = \frac{\text{maximum coefficient}}{\text{the sum of coefficients}}$$

The DRp belongs to $[0, 1]$, if the DRp is close to 1 then the weight of the sum is carried by the maximum element, this means that the data are concentrated in only one point which is the maximum coefficient of the projection. However, if the ratio is close to 0, the coefficients in the projection are well distributed which gives a cloud of peaks more significant. So, the projection with the DRp lowest is chosen as the "R θ -signature". In our example, since the projection Fig.2 (c) satisfies the two conditions, it is the "R θ -signature" of the object in figure 2 (a).

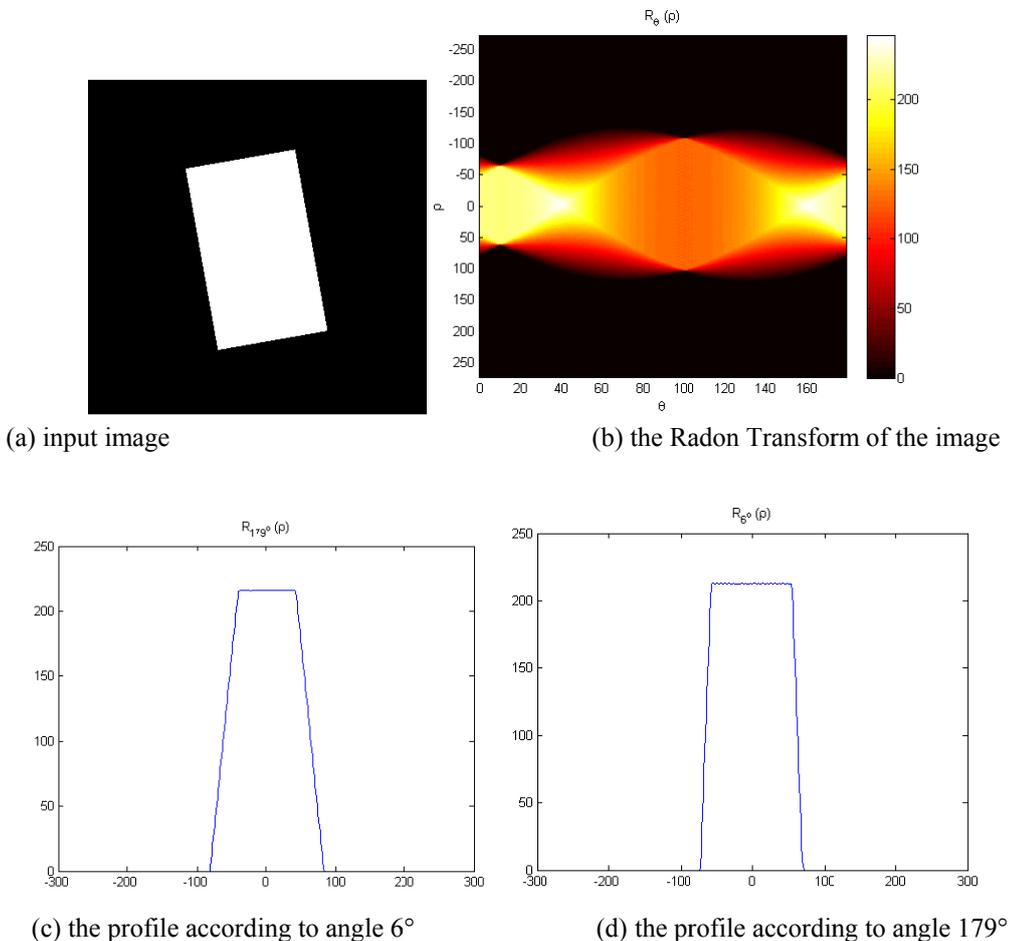


Fig. 2: The work flow to generate the R θ -signature

By generating our new signature (R θ -signature), we provide global information of a binary shape, whatever its form is. One can wonder why a binary shape, in fact, the principal operation of the RT in the discrete way (image) is the summation of the intensity of pixels along the same line for each projection. To obtain an outcome that reflects only the shape, the object must have a unique colour. Otherwise the result of RT reflects the brightness of the object in addition to its shape. For that, we will use binary images to simplify the calculation.

3.2. Proprieties of R θ -signature

Our signature preserve the proprieties of the RT (rotation, translation and scaling in Fig.3) cause our method do not interfere in the calculation of the RT but in the way of reading the data of the Radon space. It thus has three essential properties that have been mentioned in the introduction.

In addition, the signature proves to be an excellent shape measurement and it gives very good results with solid symbols (i.e. no contour or an empty symbol). Otherwise, we observe that the signature tends to compress the form so compress the signature (Fig.3(d)) and that refer to the less of information to sum when we use the contour only.

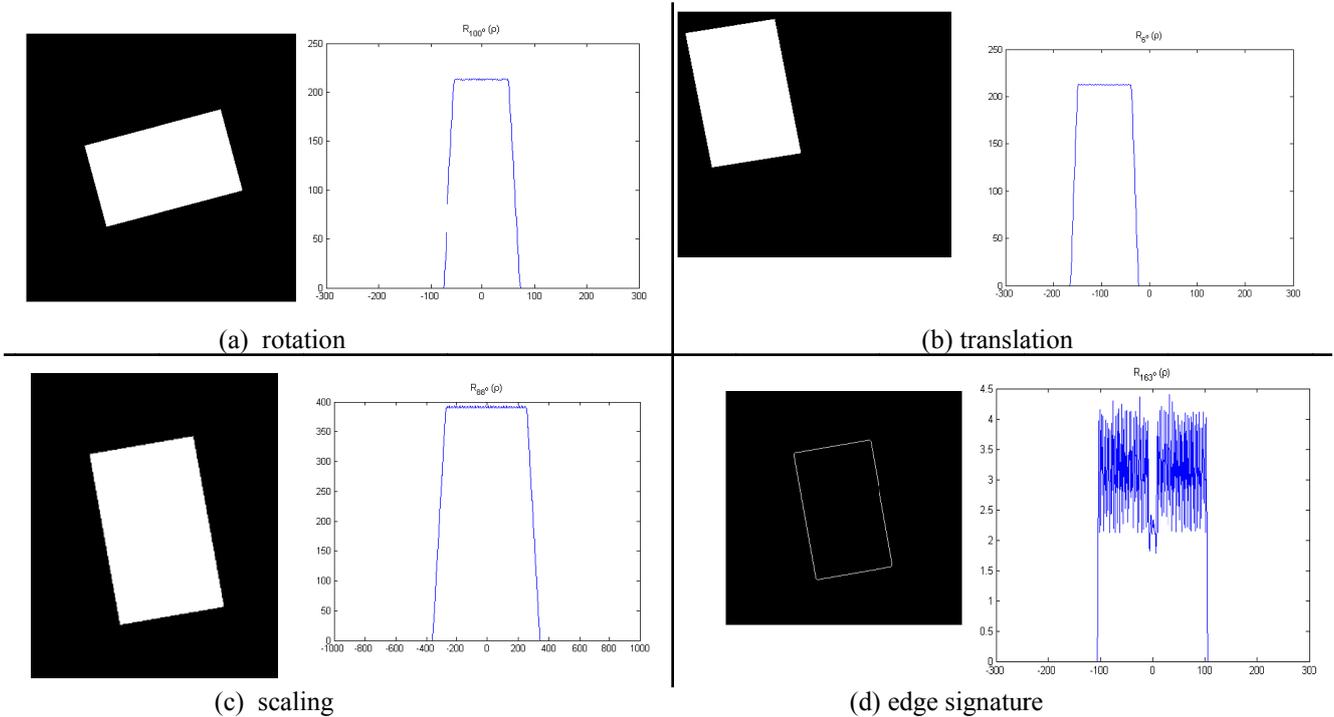


Fig. 3: The proprieties of R θ -signature

Various kinds of noise have been applied to test the robustness of our approach. One of the advantages of the proposed approach is its robustness to additive noise. This robustness is inherited from the RT itself.

Table 1 presents the variations between the original signature and the degraded signatures according to border noise, in terms of Root Mean Square error computation. Also we compared with results of R-signature of Tabbone [7]. The small differences obtained show the robustness of our approach.

4. Evaluation

We evaluate our descriptor by applying it on database of images where we try to sort those pictures by put the nearest object to the rectangular form put at first and so on. We compare its figures classification result to classification based on the R-signature [2] (given in Fig.4), our results are showed in Fig.5

Table 1: Distances between R θ -signature & R-signature of original and degraded images with increasing noise.

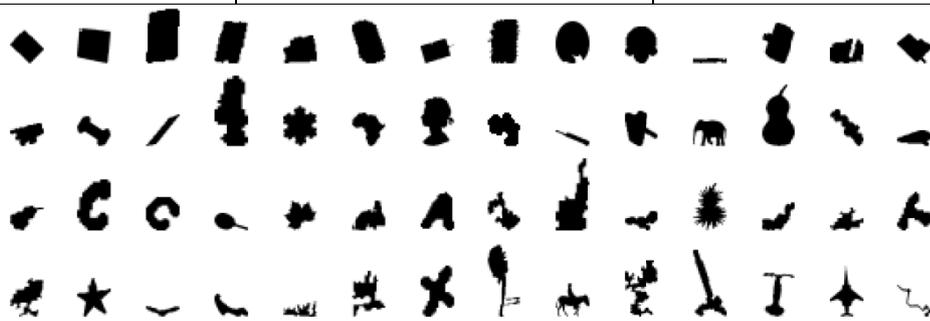
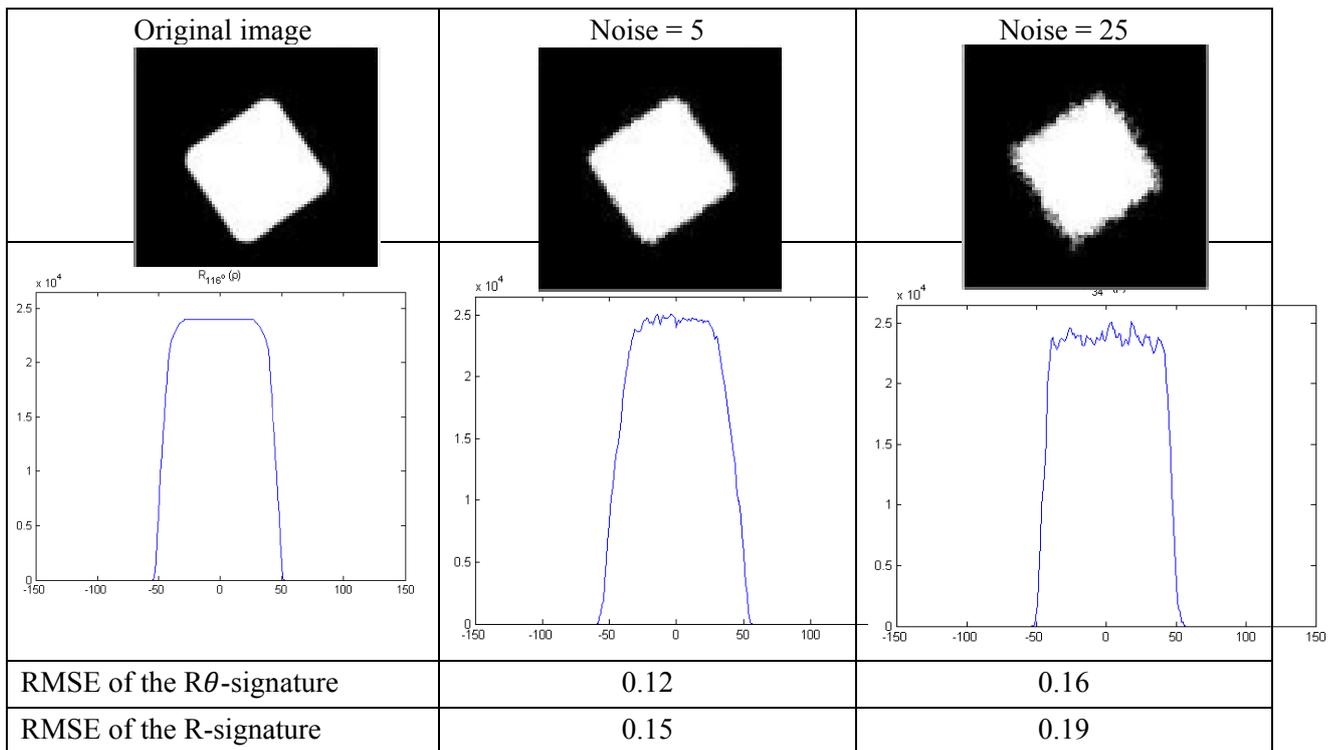


Fig. 4: The classification of the database of images using the rectangularity measurement based on R-signature

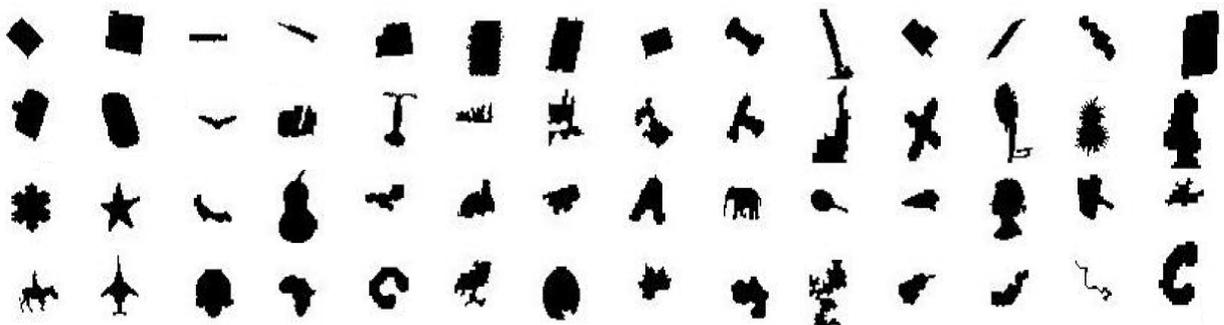


Fig. 5: The classification of the database of images using the rectangularity measurement based on $R\theta$ -signature

The arrangement of the images by the $R\theta$ -signature reveals a discrimination of the rectangular shape. The first 20th first figures have a rectangular form. A comparison between the results of R-signature and $R\theta$ -signature are illustrated in Table 2.

Table 2 reflects that our descriptor is able to discriminate the rectangular shapes from others forms since it improves the rank of rectangular shapes and disapproves that of other forms compared to other classifications. This means that our signature is a good shape descriptor.

Table 2: Comparison of the results of classification

Images	R-signature rank	Rθ-signature rank
Face1	10	45
Oval shape	9	49
Face 2	21	40
Tree	18	28
bunny	34	34
Pear	26	32
Noised rectangle 1	16	9
Noised rectangle 2	17	12
Noised rectangle 3	36	22
Noised rectangle 4	37	24
Noised rectangle 5	48	21
Noised rectangle 6	53	10

5. Conclusion

This study shows that the R θ -signature can be of great interest to differentiate and classify between graphical symbols. What makes this metric better in comparison with the standard method (MBR) is the fact that the proprieties of R θ -signature inherited from the Radon transform overcame the problems of geometrical transformations. The computation of such a feature is fast low complexity. Also our signature has the proprieties of 1D signal which means that we lower the complexity of the 2D image. To achieve more accuracy, further works will be devoted to use our signature in a building extraction application.

6. References

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