The Reference Signal Equalization in DTV based Passive Radar

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Abstract. This paper considers the problem of the co-channel interference existing in digital television (DTV) based passive radar system, which leads to the decrease of the cancellation gain and the target location difficulty caused by the false correlation peaks in the cross ambiguity function. To solve this problem, we propose two equalizers which have satisfactory effect and low computational complexity. Moreover, they can be used in the signal model where the fractional time delay is considered. Simulations verify the efficiency of the proposed methods.

Keywords: Passive Radar, Signal Frequency Network, Equalizer

1. Introduction

Passive radar is a type of bistatic radar system which has attracted increasing attention both at home and abroad because of its ability to resist the four threats[1][2]. Nowadays, with the development of DTV, the DTV based passive radar has become a new hot research area[3][4].

The single frequency network (SFN) configuration is widely used in DTV system[5]. The equalization problem of the SFN based passive radar system is a conventional key research area. There are several kinds of equalizers, such as the spatial filter, the frequency-domain equalizer, and the time-domain filter. But the spatial filter has high requirement of the antenna resolution and the frequency-domain equalizer causes the decrease of the SNR. There are also several time-domain equalizers, which share the common weakness of high computational complexity and hardware requirement. So a new equalizer is needed which has low computational complexity and easy to be realized. In order to approximate the practical situation, it is necessary to consider the fractional time delay between the co-channel interference and the direct signal.

2. The Fractional Time Delay based Signal Model in Passive Radar

Suppose that there are two transmitters (S1 and S2) in SFN, and the model of this system is shown in Fig.1.
Then suppose that the signals received by the reference antenna, which are transmitted by S1 and S2, are \( s(n) \) and \( s'(n) \), respectively. The signals of the reference channel can be given by

\[
y(n) = s(n) + b \cdot s'(n) + n_s(n)
\]

where \( n_s(n) \) is the channel noise. \( b \) denotes the amplitude attenuation and the phase shift of the co-channel interference \( s'(n) \) compared with the direct signal \( s(n) \). \( \tau \) is the time delay between \( s'(n) \) and \( s(n) \).

As a matter of fact, because of the different propagation paths, \( \tau \) may be not an exact integer multiple of the sampling period. So, if the integer multiple of the sampling period is used to show the actual time delay, the efficiency of the equalizer would be influenced. The fractional time delay is depicted in Fig.2. Here, \( T \) shows the sampling period, and \( \tau \) denotes the fractional time delay.

3. The Sinc Interpolative Method based Equalizer

In order to improve the accuracy in estimating the time delay of the multipath signal compared with the direct signal, Smith and others proposed the signal interpolative technology[6]. It shows that the signal, which has the fractional time delay, can be represented by convoluting a sinc function with the signal itself as follows

\[
s'(n) = \sum_i g(i) \cdot s(n-i)
\]

where \( g(i) = \text{sinc}(p-i) \) is the interpolative kernel function with \( p \) as the time delay of the co-channel interference after sampling.

If \( K \) is used to show the integer part of \( p \), defined by \( K = \text{fix}(p) \), then the range of \( i \) is \((-\infty, +\infty)\). It is obvious that the more the number of \( i \), the closer the signal defined by (2) gets to the real signal. In the meantime, the computational complexity increases with the increase of the number of \( i \). Luckily the value of the interpolative kernel function decreases when \( i \) increases, thus the interpolative kernel function can be cut off without losing much accuracy.

The autocorrelation function can be used to find out the time delay of the co-channel interference, which is given by

\[
f(m) = \sum_{n=0}^{\infty} y(n) \cdot y^*(n-m)
\]

The number of \( i \) is selected as 4 to explain the sinc interpolative method based equalizer.

With the precision controlled, \( y(n) \) can be approximated by

\[
y(n) = s(n) + A_{k-1} s(n-K+1) + A_k s(n-K) + A_{k+1} s(n-K-1) + A_{k+2} s(n-K-2) + n_s(n)
\]

where \( A_{k-1} = b \cdot g(K-1) \), \( A_k = b \cdot g(K) \), \( A_{k+1} = b \cdot g(K+1) \), \( A_{k+2} = b \cdot g(K+2) \). The signal defined by (4) will get peaks at positions \( 0, K-1, K, K+1 \) and \( K+2 \) after the autocorrelation function shown in (3).

When the sampling frequency is equal to the signal bandwidth, the signals given in (4) is independent to each other. So, we obtain the equation as

\[
E\left(\frac{f(K-1)}{f(0)}\right) = \frac{A_{k-1}}{1 + A_{k-1}^2 + A_k^2 + A_{k+1}^2 + A_{k+2}^2} = \frac{1}{\text{SNR}}
\]

Then, \( E(Q(K)) = m_2 \), \( E(Q(K+1)) = m_3 \) and \( E(Q(K+2)) = m_4 \) can also be written up in the same way.

The solution of the equations is
\[
A_{k-1} = \frac{1 \pm \sqrt{1 - 4 \cdot m_1 \cdot B \cdot (1 + \frac{1}{\text{SNR}})}}{2 \cdot B}.
\]
\[
A_k = \frac{m_2}{m_1}, \quad A_{k+1} = \frac{m_3}{m_1}, \quad A_{k+2} = \frac{m_4}{m_1}.
\]

where \(B = m_1 + \frac{m_2^2 + m_3^2 + m_4^2}{m_1}\). \(A_{k-1}\) can be confirmed by \(|A_{k-1}| < 1\).

An IIR filter for suppressing the co-channel interference is given by

\[
H(Z) = \frac{1}{1 - \left(-\hat{A}_{k-1}Z^{-k-1} - \hat{A}_kZ^{-k} - \hat{A}_{k+1}Z^{-(k+1)} - \hat{A}_{k+2}Z^{-(k+2)}\right)}
\]

where \(\hat{A}_{k-1}, \hat{A}_k, \hat{A}_{k+1}, \hat{A}_{k+2}\) are the estimated value of \(A_{k-1}, A_k, A_{k+1}, A_{k+2}\), respectively.

4. The Weiner Filter based Equalizer

Although the sinc interpolative method based equalizer is effective and easy to be realized, it is not a common phenomenon that the sample rate equals the signal bandwidth. The sample rate is larger than the signal bandwidth most of the time, then (5) can hardly be used any more. Another method should be proposed to evaluate \(A_{k-1}, A_k, A_{k+1}, A_{k+2}\).

We find that if the reference channel signal shown in (1) is used as the expected response of the Weiner filter, the weight vector can be used to construct the IIR filter defined by (7). Equation (1) has the most obvious peaks at 0, \(K\) and \(K+1\) after the autocorrelation function. Then a new signal is constructed as the input of the Weiner filter

\[
x(n) = s(n-k) + b \cdot s'(n-k) + n_s(n)
\]

where \(k\) is an integer number less than \(K\). \(s(n-k), b \cdot s'(n-k)\) and \(n_s(n)\) are the direct wave, the co-channel interference and the noise delayed, respectively. \(R_{ss}\) is the autocorrelation matrix of the tapped input matrix of the filter.

The signal defined in (1) is used as the expected response of the Weiner filter. \(r_{xy}\) is the cross-correlation between the tapped input matrix and the expected response.

The two noise signals shown in (1) and (8) are irrelevant, so they only affect the autocorrelation matrix of the input signal. The co-channel interference, however, can affect both \(R_{ss}\) and \(r_{xy}\):

\[
R_{ss} = R_{ss} + R_{ss'} + R_{nn} = R_{ss} + R_{ss'} + \sigma^2 I
\]

where \(R_{ss}\) and \(\sigma^2\) are the autocorrelation matrix and the power of the noise \(n_s(n)\), respectively. \(R_{ss'}\) represents the autocorrelation matrix of \(s(n-k)\). \(R_{ss'}\) is the added matrix of the autocorrelation matrix of the co-channel interference. So, when there are co-channel interference and noise in the input signal and expected response of the filter, the optimal solution of the Weiner filter is

\[
w = (R_{ss} + R_{ss'} + \sigma^2 I)^{-1} r_{xy}
\]

where \(w\) is the biased estimate of the real system. Since this, we propose the two-stage filter to get more accurate weight vector. The system diagram of the Weiner filter based equalizer is shown in Fig.3.

![Fig.3 The system diagram of the Weiner filter based equalizer](image)

The input signal of the second-order filter is defined as

\[
x'(n) = e(n-k)
\]

where \(e(n-k)\) is delayed by \(k\) compared with the output signal of the first-order Weiner filter \(e(n)\).

\(y(n)\) is used as the expected response of the second-order filter. Equations (8) and (10) show that there are new co-channel interference \(s'(n-k)\) and \(s'(n-k-k)\) in the output signal of the second-order Weiner filter, so it can’t be used as the estimated direct signal.
The weight vector $\mathbf{W}(n)$ of the second-order Weiner filter has relatively accurate estimate of $A_{k-1}$, $A_k$, $A_{k+1}$, $A_{k+2}$, so we can use it to construct the IIR filter defined by (7).

5. Simulation and Discussion

The simulation conditions of the two equalizers are set as follows. The signal from S2 is delayed by 100.3 sampling period compared with the signal from S1. In the Weiner filter method based equalizer, $k$ defined in (8) is 99. Both the order of the first-order filter and the second-order filter are 4. The cancellation gain is defined as

$$G = 20 \times \log_{10} \left( \frac{\text{the peak value of the 100 time delay before filtered}}{\text{the peak value of the 100 time delay after filtered}} \right)$$

(12)

Fig.4 shows the cancellation gain of the two equalizers under different parameters. The autocorrelation of the reference signal in different conditions is shown in Fig.5. We can see the co-channel interference is filtered off and the secondary peak of the autocorrelation of the reference signal is down over 30dB with the two equalizers.

6. Summary

Two reference signal equalizers in the SFN based passive radar are proposed in this paper, they have good performance in the suppression process of the co-channel interference. In order to make the algorithms approach to the practical application, the fractional time delay is in consideration to set up the signal model. After analyzing the signal model of the SFN based passive radar, two algorithms are proposed, and then the simulation verification is shown to prove the efficiency of the algorithms.

Fig.4: The cancellation gain of the two equalizers with different parameters
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8. References