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# Analytical Study of MC-CDMA-TCM over Multi-path Rayleigh Fading Channels

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**Abstract.** The exponential growth of multimedia users demand fast data rates and reliable transmission which leads us towards fourth generation (4G) wireless technology. In recent years, the adaptive modulation based multi-carrier code division multiple access (MC-CDMA) systems captured maximum attention. In this paper MC-CDMA-TCM is tested over wireless fading channel. The objective of this research is to perform an analytical study of the MC-CDMA-TCM system to achieve theoretical BER curve. A conclusion is drawn on the basis of the outcomes of research which favors the system for better performance over Rayleigh fading channel but the increasing design complexity limits the system to use very high constellation orders.

**Keywords:** MC-CDMA, Rayleigh fading channel

## 1. Introduction

With ever-increasing growth of user demand, the emergence of new mobile broadband technologies wireless services have been going at a rate greater than 50% per year [1]. Many prophetic visions have appeared in the literature presenting 4G as one of the boundaries of wireless mobile communication. 4G technology allows user to efficiently share common resources, whether it involves the frequency spectrum, computing facilities, databases, or storage facilities. One of the important objectives of the 4G wireless systems is to consider severe inter symbol interference (ISI) resulting from the high data rates, hence utilizing available limited bandwidth in a spectrally efficient manner. To achieve these objectives, there are two principle contending technologies, i.e. Orthogonal Frequency Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA). CDMA is a well-known standard and has been used for several years where else OFDM, a multi-carrier (MC) technique, is relatively new. OFDM is represented as the successor to frequency hopping and direct sequence CDMA. It is also positioned to support next generation wireless LANs and metropolitan networks. One of the qualities of OFDM is to mitigate multi-path distortion in a spectrally efficient manner without the requirement of multiple matched filters that has won adherents in the IEEE 802.11a and IEEE 802.16 working groups [2]. Future 4G wireless systems, based on the combination of multi-carrier (OFDM) and spread spectrum (CDMA) technologies, popularly known as OFDM-CDMA (or MC-CDMA), is applied to a wide-area environment. This combination of OFDM and CDMA can also achieve very large average user throughputs. The benefit of using multiple sub-carriers is that, because each carrier operates at a relatively low bit rate, the duration of each symbol is relatively long. Hence if a million bits per second are sent over a single base-band channel, the duration of each bit will be under a microsecond. This imposes severe constraints on synchronization and removal of multi-path interference. If the same million bits per second are spread among N sub-carriers, the duration of each bit can be longer by a factor of N, and the constraints of timing and multi-path sensitivity are relaxed.

Combining multi-carrier OFDM transmissions with code division multiple access (CDMA) allows us to utilize the wideband channel's inherent frequency diversity by spreading each symbol across multiple sub carriers. Prasad and Hara [3] compared various methods of combining the two techniques. They proposed three different structures, namely multi-carrier CDMA (MC-CDMA), multi-carrier direct sequence CDMA (MC-DS-CDMA) and multi-tone CDMA (MT-CDMA). Coded OFDM-CDMA with adaptive modulation is considered as an efficient technique for 4G mobile systems, in which the received signal is normally interfered by multi path effects [4]. Trellis Coded Modulation (TCM) [5, 6] was originally designed for transmission over Additive White Gaussian Noise (AWGN) channels, where it is capable of achieving a coding gain without bandwidth expansion. Turbo TCM (TTCM) is a bandwidth efficient transmission scheme, which has a structure similar to that of the binary turbo codes distinguishing it by using TCM schemes as component codes [5, 6]. Both the TCM and TTCM schemes employed set partitioning based signal labeling, in order to increase the minimum Euclidean distance between the encoded information bits.

This paper investigates the Mathematical Model of a Trellis Coded Modulated MC-CDMA system with TCM over wireless fading channels. Specifically, two ray multi-path channel employing Jakes model is assumed. Meaningful bit error rate (BER) performance curves are generated based upon the result, useful comparisons are extracted and design guidelines are provided for the upcoming 4G communication systems.

## 2. System Description

A random PN Sequence of zeros and ones with spreading factor of 'L' chips per bit is generated to spread the incoming bit stream. The binary sequence is selected due to the fact that a TCM Encoder expects the frame based binary data. The output of the spreader is wideband signal. The dimensions of the output will be 'L' times the input data bits. The wideband binary sequence is fed to the TCM Encoder.

For our model [7] as shown in the fig. 1, most of the simulations are carried out with 8-PSK signal constellation the convolutional encoding will be carried out at the rate of ½. To achieve multi carrier modulation the OFDM block is used. The input to the block is the complex output from the TCM Encoder. In this model N sub-carries are assumed. Transmitters are usually placed in changing environment surrounded by reflecting objects and scatterers that prevent the existence of a line of sight between the transmitter and the receiver. Under the assumption that there are many local objects surrounding the mobile terminal, the received signal will consists of a sum of many signals that arrive unequally attenuated and with different angles at the receiver. The channel creates frequency selective fading on the received signal. To model this in simulations a Finite Impulse Response (FIR) filter with time varying taps is commonly used. Consider a frequency selective fading channel model as a tapped delay line, the low pass equivalent impulse response is given by [8].

$$h(\tau;t) = \sum_{p=1}^{P} h_p(t)\delta(t-\tau_p) \tag{1}$$

where P is the total number of paths, τp is the propagation delay for the pth path and hp(t) is the complex envelope of the signal received on the pth path. The effect of white Gaussian noise is also considered as n(t) which will be added to the faded signal. For the proposed model a Two-ray Rayleigh multi-path fading channel is selected for indoor environment. By convention, the first delay is typically set to zero. The first delay corresponds to the first arriving path. The ability of a signal to resolve discrete paths is related to its bandwidth. If the difference between the largest and smallest path delays is less than about 1% of the symbol

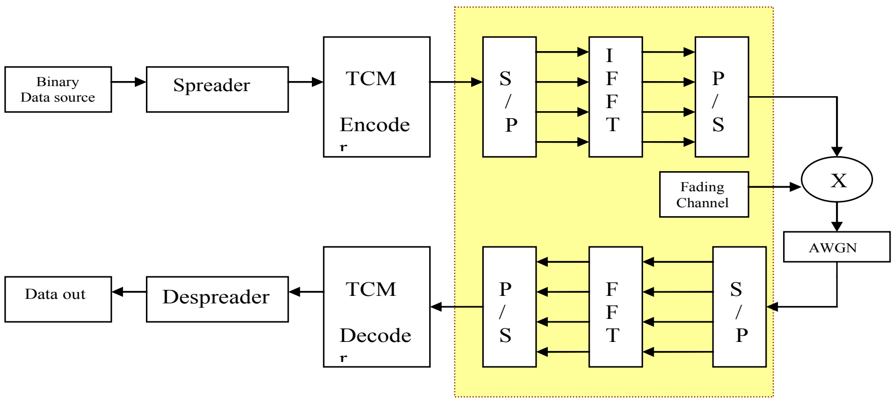


Fig. 1: Block Diagram of the system

period, then the signal experiences the channel as if it had only one discrete path. The average path gains in the channel object indicate the average power gain of each fading path. In practice, an average path gain value is a large negative dB value. The dB values in a vector of average path gains often decay roughly linearly as a function of delay, but the specific delay profile depends on the propagation environment. The expected value of the path gains' total power is 1, i.e. path gains for the system are normalized.

The relative motion between the transmitter and receiver causes Doppler shifts. The maximum Doppler shift corresponds to the local scattering components whose direction exactly opposes the mobile's trajectory. This parameter defines the speed of fading channel. If the value of $f_d$ is zero then the channel is static, otherwise between the range of 4Hz to 100Hz or may be even more it can be characterized as Slow or Fast fading Channel. In this system $f_d$ = 40Hz. The reverse operation is performed assuming the perfect channel estimations at the receiver side. With the same specifications of the trellis structure defined at transmitter, the decoder demodulates the symbols into the coded bits and recovers the original data bit stream by deploying Viterbi Algorithm for decoding the convolutionally encoded data. Table 1 presents the system specifications that are taken into account for simulation. The bit rate is kept constant at 10Mbps.

Table 1: System Specifications

| No. of Careers | L | M | FFT Size | Path delays (ns) | Path gains(dB) | $f_d$ (Hz) | Encoder Rate |
|---|---|---|---|---|---|---|---|
| 32 | 15 | 8 | 256 | 0, 25 | 0.97, -7 | 40 | 1/2 |

# 3. System Analysis

Let ‘K’ active users use the MC-CDMA system at the same time instant‘t’. Each user k transmits ‘n’ data bits of duration ‘$T_b$’. Where $d[n] \in [0,1]$, $k \in \{1, 2, 3, \ldots K\}$. The data bits of user ‘k’ are first sequentially spread into ‘L’ chips. A random PN sequence code, $\sum_{i=0}^{L} G_k[l]$ and $L \in \{0, 1, 2, \ldots L\}$ is used. Hence $G_k[l]$ denotes the spreading code of length L employed by user k. The chip duration $T_c$ is related to the bit duration Tb according to the equation 2 as

$$T_b = L\,T_c \tag{2}$$

In MC-CDMA transmission, the carier bandwidth- $B_c$ is divided into ‘N’ sub-cariers of Bandwidth $B_{sc}$ so that,

$$B_{sc} = B_c / N \tag{3}$$

Hence every sub-carier is orthogonal by a factor of $1/T_b$ or has the centre frequency at $f_0 + k / T_d$, where $f_0$ is the lowest centre frequency of sub-carrier.

Now the spread signal is:

$$S_k(t) = \sum_{n=-\infty}^{+\infty}\left[\left[d_k[n]\left(\sum_{l=0}^{L} G_k[l]\right)\right]p(t - nT_c)\right] \tag{4}$$

where p(.) is the unit amplitude pulse. If we take the time independent spreaded signal, then

$$X^i[n] = d_k[n] \oplus G_k[l],\ for\ \ l = 0,......L-1 \tag{5}$$

where $\oplus$ symbol showing XOR operation which is as follows

$$X^i[n] = \urcorner\left(\left(d_k[n] \wedge G_k[l]\right) \vee \left(\urcorner d_k[n] \wedge \urcorner G_k[l]\right)\right) \tag{6}$$

A set of two bits, each from incoming chips, is fed to TCM encoder as shown in fig. 2 and can be written as

$$X^i = [x_o\, x_1\,,\, x_2\, x_3\,,\, x_4\, x_5\,, \ldots\ldots..] \tag{7}$$

i.e. $x_o\, x_1 = X^0$, $x_2\, x_3 = X^1$

Equation 7 can be re-written as

$$X^i = [X^0, X^1, X^2, \ldots\ldots] \,;\ 0 \le i \le \infty \tag{8}$$

One of them is uncoded and other is an input to ½ rate convolutional encoder, it can be given as

$$Y^i_{EN} = [x_0 c_1 c_2, x_2 c_3 c_4, \ldots\ldots\ldots.] \tag{9}$$

or

$$Y^i_{EN} = [Y^0, Y^1, Y^2, \ldots\ldots..] \tag{10}$$

If the data is binary then $Y_{EN}$ must have one of the from

$$Y^I_{TCM} = [Y^0_{TCM}, Y^1_{TCM}\, Y^2_{TCM}, \ldots\ldots., Y^7_{TCM}] \tag{11}$$

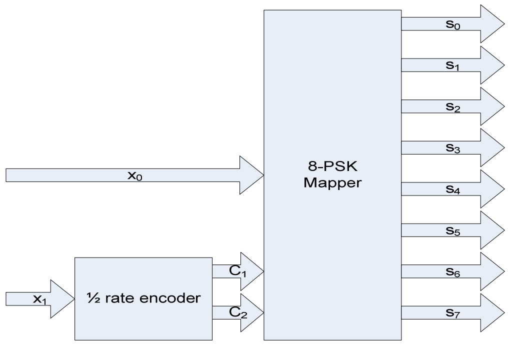


Fig. 2. Internal Block diagram of TCM Encoder

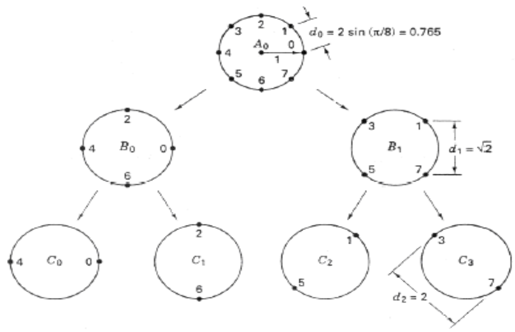


Fig. 3. Set partitioning of 8 PSK

= [000, 001, 010, 011, 100, 101, 110, 111]

The output of TCM defines by the symbol mapping which must be done on the basis of the Ungerboek [6], set partitioning method to ensure the redundancy. Since we are working on the 8-PSK TCM that is:

Now as per set partitioning theory this large constellation will be partitioned into two subsets i.e. $B_0$, $B_1$. Then $B_0$ and $B_1$ are further split into $C_0$, $C_1$, $C_2$, $C_3$ as shown in the figure.3 [9] If we take a single set of encoder output with uncoded incoming bit as,

$$Y_{EN}^{i} = Y_{EN}^{0} = y_0^0 y_1^0 y_2^0 \rightarrow x_0 c_1 c_2 \tag{12}$$

Let $y_0^0$ is the uncoded bit then, If $y_0^0 = 0$ then $Y_{EN}^{i} \in B_0$ and $B_0 = \{c_0, c_1\}$, If $y_1^0 = 0$ then $Y_{EN}^{i} \in C_1$ and $C_1 = \{s_2, s_6\}$ and $s_2 = Y_{TCM}^{0}$, $s_6 = Y_{TCM}^{1}$ otherwise $Y_{EN}^{i} \in C_0$ and $C_0 = \{s_2, s_4\}$, If $y_2^0 = 0$ then $Y_{EN}^{i} = s_2$ otherwise $Y_{EN}^{i} \in s_6$

Similarly for the reverse case: $$Y_{EN}^{i} = y_0^0 y_1^0 y_2^0 = Y_{EN}^{0} \tag{13}$$

now, if $y_0^0 = 1$ then $Y_{EN}^{i} \in B_1$ and $B_1 = \{c_2, c_3\}$ and if $y_1^0 = 1$ then $Y_{EN}^{i} \in C_3$ and $C_3 = \{s_3, s_7\}$ otherwise

$$Y_{EN}^{i} \in C_2 \text{ and } C_2 = \{s_1, s_5\} \tag{14}$$

If $y_0^2 = 1$ then $Y_{EN}^{i} = s_7 = Y_{TCM}^{7}$ Otherwise $Y_{EN}^{i} = s_3 = Y_{TCM}^{6}$

Form above mathematical algorithm it can be clearly seen that outcome of TCM [9] merely depends upon the uncoded bit. The IFFT of the encoded bits [7] can be expressed as shown in Equation 15.a. Since $Y_{EN}^{i}$ is a set of 3 bits and $f_m$ is the frequency of the $m^{th}$ sub-carrier. In a channel, it is assumed that k users are sending their signal asynchronously with delay τ ($0 \le \tau \le T_b$). The transmitting signal is now shaped as shown in Equation 15.b:

$$y_k(t) = \sum_{i=0}^{\infty} \sum_{m=0}^{N-1} Y_{EN}^{i} e^{j2\pi f_m (t - T_d)} \tag{15.a}$$

$$S_k(t) = \sum_{i=0}^{\infty} y_k(t) p\left(t - \frac{i}{3} T_c\right) \tag{15.b}$$

The impulse response for $k^{th}$ user then;

$$h_k(t) = \sum_{p=1}^{p} \alpha_p \delta(t - \tau_p) \tag{16}$$

Where $\alpha_p$ is the path gain of $p_{th}$ path and is also considered constant for the one bit duration. The received signal is then given by:

$$R_k(t) = [S_k(t) * h(\tau_{0,} t)] + n(t) \tag{17}$$

## 3.1. Interference Analysis

Since a multi-carrier system is considered with multi-path fading channel hence signal can be interfered by its delayed version as well as by the signal at other sub-carrier.

$\gamma_p^{l=m}$ = Interference from same sub-carrier

$\gamma_p^{l\neq m}$ = Interference from other sub-carrier at $p^{th}$ path

If λ is the desired signal received at direct path [10] then $\lambda = \alpha_{LOS} y^i[n]N$ where $0 \le i \le \infty$ (18)

For every individual value of 'i', 0 ≤ n ≤ 2 and $\alpha_{LOS}$ is path gain at line of sight, $\lambda_p^{l=m}$ is desired signal component of delayed path'p' with the same sub-carrier. The delayed signal can aid the better reception of the signal as its contents are same as that of λ. Since the desired signal is assumed as Gaussian random variable hence the mean of delayed version can be given as [10].

$$E\left[\lambda_p^{l=m}\right] = \sum_{l=0}^{N-1} \frac{2\alpha_p}{T_b} y^i[n] R^{l=m} \quad \text{where } R^{l=m} = \frac{1}{2}\left(T_b - \tau_p\right) Cos 2\pi f_m \tau_p \tag{19}$$

and the variance is given as
$$Var\left[\lambda^{l=m}\right] = \left[\sum_{l=0}^{N-1} \frac{2\alpha_p}{T_b} L^{l=m}\right]^2 \quad \text{where } L^{l=m} = \frac{1}{2}\tau_p Cos\left(2\pi f_m \tau_p\right) \tag{20}$$

Now the total interference of the system [10] can be given as

$$\zeta = \left[\sum_{p=1}^{P} \left\{Var\left[\gamma_p^{l=m}\right] + Var\left[\gamma_p^{l\neq m}\right]\right\}\right] + \sigma^2 \tag{21}$$

Where $\sigma^2$ is the variance of random Gaussian noise [10]

$$Var\left[\gamma_p^{l=m}\right] = \frac{N(K-1)\sigma_p^2}{3} \quad \text{and} \quad Var\left[\gamma_p^{l\neq m}\right] = \frac{(K-1)\sigma_p^2}{2\pi^2} \sum_{l=0, l\neq m}^{N-1} \sum_{m=0}^{N-1} \frac{1}{(l-m)^2} \tag{22}$$

$\sigma_p^2$ is the power profile of the signal at $p^{th}$ path. Now the signal strength can be given as the ratio of useful signal to the interfering components.

$$SNIR = \frac{\left(\lambda + \sum_{p=2}^{P} E\left[\lambda_p^{l=m}\right]\right)^2}{\zeta} \quad \text{and} \quad BER = \frac{1}{2} erfc\left(\sqrt{SNIR/2}\right) \tag{23}$$

## 4. Result and Discussion

In this section the efficacy of the analysis is verified by simulating system model of figure 1 with parameters given in table 1. It can be seen in figure 4 that for lower SNR values simulated curve shows different response when compare with analytical results. But at high SNR values it is simply the replica of theoretical results. The reason is that the estimated channel is assumed for the simulation.

The same system is simulated with different constellation sizes, number of memory elements and encoding rates.

It is observed that the system which performs well in all aspects, prove to be most complex. The complexity factor (CF) of various systems is calculated by [11] with the number of memory elements 'υ' deployed and given in table 2. It is clear from the table that complexity factor is proportional to the measure of reliable communication provided by the system. Since TCM decoders deploy Viterbi Algorithm which uses most likelihood (ML) sequence to decode the encoded message. Infinite memory elements are required to keep all previous states for error free decoding [12] which is practically not possible and the reliability of

Table 2: System observations and measurements

| **M-array** | 8 PSK | 8 PSK | 16 QAM | 16 QAM |
|---|---|---|---|---|
| **υ** | 3 | 7 | 3 | 7 |
| **CF** | 11.313 | 181.01 | 12.69 | 15.79 |

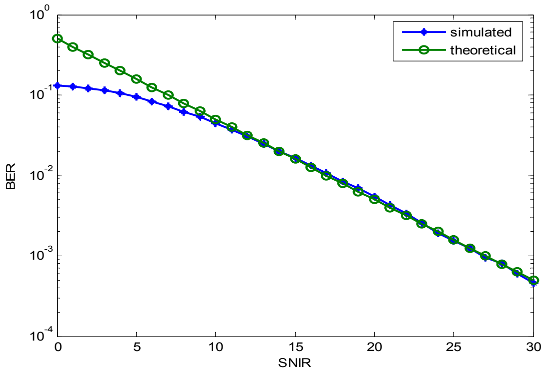


Fig. 4. Theoretical vs simulated results

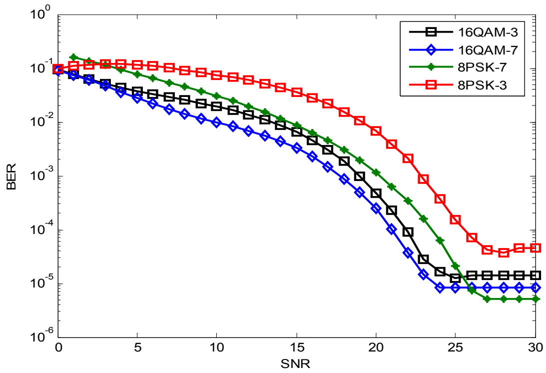


Fig. 5. System performance with different constraints lengths and constellation order

received data is therefore affected. It also limits the increase of constellation size at the transmitter to avoid complex receiver design.

## 5. Conclusion

From the results it is concluded that simulated results are close to the theoretical results, especially for the higher values of SNIR. Best performance is shown by the system with maximum memory elements and high constellation orders. The increasing complexity factor of the system design puts limit on excessive increase in such parameters.

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