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Diversity-Rate Considerations and Precoder Designing for Spatially Correlated Perfect Space-Time Block Codes

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Abstract. Precoders are designed at the transmitter to compensate the antenna correlation effect when space-time coding is used. In this paper, a linear precoder is designed for a new class of non-orthogonal space-time block codes called perfect space-time block codes (PSTBC). The word "perfect" stands for some remarkable characteristics such as full diversity, full rate and non-vanishing constant minimum determinant for increasing spectral efficiency. The performance of the system is degraded in the spatially correlated channel condition, therefore, we investigate the effect of channel correlation on diversity and coding gain of PSTBC. Furthermore, the precoder is calculated based on the knowledge of the matrix of correlation and the channel mean in order to minimize the pair-wise error probability (PEP) criterion. Simulation results demonstrate that using the proposed precoder improves the performance of PSTBC system in correlated channels.

Keywords: correlated channel, division algebras, PEP criterion, perfect space-time block code, precoding.

1. Introduction

Recent advances in wireless communications have dramatically increased the performance and reliability of wireless systems and different approaches have been proposed for this progress. One of these concepts is multiple-input multiple-output antennas (MIMO) structure in which space-time coding that can be used to achieve higher performance, throughput and coding gain and Space-time block coding (STBC) was first introduced by Alamouti [1] and Tarokh [2],[3].

Perfect STBC is a class of non-orthogonal STBC that satisfies full rate, full diversity and non-vanishing constant minimum determinant properties for increasing spectral efficiency[4]. No orthogonal STBC schemes exist that can satisfy both full-diversity and full-rate transmission features. In other words, there is a fundamental tradeoff between these two gains [10]. On the other hand, The code design criteria in [2] and [3] assume that the transmit and receive antennas are uncorrelated. This may not be accurate in practical situations that MIMO antennas are correlated, due to the lack of spacing between them. The effect of spatial correlation on the MIMO channel capacity has been addressed in [5]. A linear precoder was calculated in [6] for orthogonal-STBC and In [7] for quasi orthogonal case. In [8] an approximated solution was given for the case of non-zero means. In our previous work [9], we investigated the performance of constructing and decoding of perfect space-time block codes in the presence of channel estimation error and multiple access interference in a multiuser MIMO downlink scheme applicable to CDMA systems.

In this paper, we investigate perfect STBC and its diversity and coding gain in the situation of channel correlation and finally we recommend a linear precoder for this system. We assume the knowledge of the transmit antenna correlation and also channel mean in the transmitter and improve the performance of a space-time coded system in this situation.

The rest of the paper is organized as follows. Section 2 describes the system model. In section 3 cyclic algebras and perfect space-time block codes are considered. In section 4 the diversity and coding gains will

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be discussed. In section 5 precoding design is defined. The simulation results are presented in section 6. Finally Section 7 concludes the paper.

2. Overview of the System Model

We consider a system, with N_T transmit antennas and are N_R receive antennas as depicted in Fig.1. In such a system, the precoder can be viewed as a processing block in addition to an existing STBC. Therefore we have $Y = \sqrt{E_S}HFC + N$ Where Y is an $N_R \times T$ matrix that includes all received signals during T time slots; X is an $N_T \times T$ matrix that includes all transmitted signals and H is the channel matrix. Also C is the coded stream and F is the precoding matrix which will be discussed later. The receiver performs maximum-likelihood (ML) detection over a codeword C to obtain

$$\hat{C} = \arg\min_{C\in\mathcal{C}} \left\| Y - \sqrt{E_S}HFC \right\|_F^2 \quad (1)$$

where $\mathcal{C}$ is the STBC codebook, and the subscript F denotes the Frobenius norm. In practice, there can be a correlation between the transmit antennas. Therefore, the channel can be modeled as below

$$H = H_m + R_r^{1/2} H_w R_t^{1/2} \quad (2)$$

Where H_w is an $N_R \times N_T$ i.i.d. complex Gaussian matrix with zero mean and unit variance, H_m is the channel mean matrix and $R_t = R_t^{1/2} R_t^{1/2H}$ is the transmit antenna correlation and $R_r = R_r^{1/2} R_r^{1/2H}$ is the receive antenna correlation. A common, simplified correlation model assumes that the correlation between the receive antennas elements does not depend on the transmit antennas and vice versa. In other words, transmit and receive correlations are separable and uncorrelated. This assumption can be justified by the fact that these arrays are sufficiently far apart with enough random scattering between them [5].

In this paper, we focus on correlation at the transmitter, and we assume no correlation in the other side of the link. Thus, the channel model in this environment is reduced to

$$H = H_m + H_w R_t^{1/2} \quad (3)$$

The receiver is assumed to know the channel perfectly, i.e. it knows the channel realization whereas the transmitter only knows the channel mean and the transmit correlation.

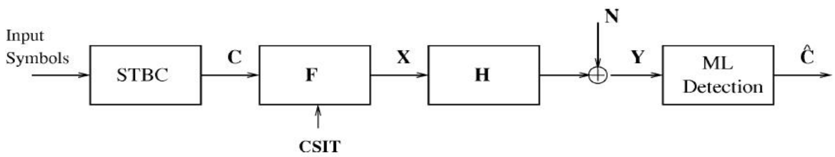

Fig. 1.System architecture with a linear precoder and STBC

3. Cyclic Algebras and Constructing Perfect Space-Time Block Code

The major structure of Perfect STBC comes from cyclic division algebra. They enable us to design high rate, highly reliable Space-Time codes, which are characterized by many optimal features. Their algebraic properties can be exploited to improve the design of good codes. Perfect space-Time block codes are defined as below [4]. A $N_T \times N_T$ STBC is called a perfect code if and only if:

- It is a full rate linear code using $N_T{}^2$ information symbols either QAM or HEX.
- The minimum determinant of the infinite code is non-Zero, so that the rank criterion is satisfied.
- The energy required to send the linear combination of the information symbols on each layer is similar to the energy used for sending the symbols themselves.
- It induces uniform average transmitted energy per antenna in all T time slots.

Given a field F, let K be a cyclic extension of F of degree n, i.e. Gal(K/F)= $\langle\sigma\rangle$ where σ is a generator of the cyclic group and a cyclic division algebra $D = (K/F, \sigma, \gamma)$ is the set of all elements with above form. Hence, for some "non-norm" element $\gamma \in F^*$, $e^n = \gamma$ and $\forall x \in K$, $e^{-1}xe = \sigma(x)$ and for $x \in D$ and $x_i \in K$. This "non-norm" choice is a key factor in designing non-vanishing determinant cyclic division algebra codes. These codes are a special class of codes which use algebraic number-theoretic constellations as component codes. In the general case of degree n, we have $x_i \in D$ and so [12]

$$X = \begin{pmatrix} x_0 & x_1 & x_2 & \cdots & x_{n-1} \\ \gamma\sigma(x_{n-1}) & \sigma(x_0) & \sigma(x_1) & \cdots & \sigma(x_{n-2}) \\ \vdots & & & \ddots & \vdots \\ \gamma\sigma(x_2) & \gamma\sigma(x_3) & \gamma\sigma(x_4) & & \sigma^{n-2}(x_1) \\ \gamma\sigma^{n-1}(x_1) & \gamma\sigma^{n-1}(x_2) & \gamma\sigma^{n-1}(x_3) & \cdots & \sigma^{n-1}(x_0) \end{pmatrix} \quad (4) \quad \text{and} \quad e = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ \gamma & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad (5)$$

And finally [9],[13]

$$X = \sum_{l=1}^{N_R} (\phi_l e^{l-1}) diag(M b_l) \quad (6)$$

Where e is defined by using (5), b_l is a complex QAM vector, ϕ_l is chosen to ensure full diversity and maximize the coding gain [14], M is defined in [17] and γ is a non-norm element of the division algebra satisfies $|\gamma| = 1$. For Perfect STBC, equation (6) is used with $\phi = 1$ and equation (5) is reconfigured with $\gamma = j$. Although [4] proposes an algorithm for constructing perfect codes for dimensions of $2\times 2, 3\times 3, 4\times 4$ and 6×6, it is shown that by exploiting some relaxation methods for the definitions, perfect space-time codes can be obtained for any number of antennas [17].

4. Diversity and Rate Considerations

It has been shown [10] that codes constructed from cyclic division algebras with non-vanishing minimum determinant achieve the Diversity vs. Multiplexing Gain tradeoff. The performance of STBC directly depends on the diversity and coding gain which are attained by the rank and eigenvalues of the code error covariance matrix [16]. When the channel is i.i.d, the maximum diversity is attained if the error matrix is full rank [2] as $rank(c - \hat{c}) = N_T$. This equation can be reformulated as [4] saying that the codebook is full diverse if $\left|\det(c - \hat{c})\right|^2_{c \neq \hat{c} \in □} \neq 0$. For maximizing the throughput the code should be full rate, for example in a four symbol constellation the four symbols in the codewords are functions of four symbols independently chosen such that $|□| = \gamma^4$. The asymptotic coding gain is given by the minimum determinant of □ as $\delta_{\min}(□) = \min_{x_i \neq x_j \in □} \left|\det(c - \hat{c})\right|^2$. A good Perfect STBC attempts to maximize $\delta_{\min}(□)$.

We consider the pairwise error probability (PEP) as the performance criterion, which is the probability that a transmitted codeword has a worse detection metric than another codeword. Solving the exact PEP is very complex. Hence, by applying the Chernoff bound, similar to [2], the PEP can be bounded by

$$P(C \to \hat{C}) \le \exp\left(- \frac{(E_s \left\| HF(C - \hat{C}) \right\|_F^2)}{4\sigma^2} \right) \quad (7)$$

The average PEP can be written as $\overline{PEP} = E_H\left(P(C \to \hat{C} \mid H)\right)$. By exploiting the Gaussian pdf and after averaging, the PEP bound is achieved which will be discussed in details in section 5. In this case, the diversity and coding gain (which depends on the rank and the product of non-zero eigenvalues of code error covariance matrix A in i.i.d situation), depends on the interaction of the error matrix and the channel correlation matrix in correlated channels. In [16] this situation is analyzed for orthogonal STBCs with the virtual channel representation [20] and proved that this interaction is governed by the property of $R(A \otimes I_{N_R})$ in which $R = E(hh^H)$ where $h = vec(H)$ and A is the error covariance matrix. In correlated channels, the diversity gain is no longer controlled by above equation and is defined as the rank of $R(A \otimes I_{N_R})$ and is bounded as

$$\text{d=rank}(R(A \otimes I_{N_R})) \le \min(\text{rank(R)}, \text{rank}(A \otimes I_{N_R})) = \min(\text{rank(R)}, N_R □ \quad (8)$$

The above expression demonstrates that the diversity gain is no longer directed by the rank of matrix A, i.e. the case of uncorrelated channels, but also that of channel covariance matrix. Using Perfect STBC and the above correlation model for the channel, we proved the same result.

5. Precoding Design

Consider the PEP with ML detection and following the algorithm proposed in [21] and exploiting it for Perfect STBCs, an effective precoder is designed. The codeword distance product matrix is defined as

$A=\frac{1}{P}(C-\hat{C})(C-\hat{C})^*$ Where P is the average transmit power and $\rho = {}^{P}\!/_{\sigma^2}$ is the SNR. The Chernoff bound can be re-written as [21]:

$$f(H,A,F)=\exp\left(-\frac{\rho}{4}tr(HFAF^*H^*)\right) \tag{9}$$

We aim to minimize the Chernoff bound on the mentioned PEP. Since the PEP is codeword-pair dependent, an appropriate design method is needed for selecting the codeword distance product matrix over which the PEP is going to be optimized [2],[6]. An optimization problem can be formulated as follows [8,21]

$$J=tr(H_m W^{-1} H_m^*)-M\ \log\det(W) \tag{10}$$

$$\text{Subject to}\quad W=\frac{\rho}{4}R_t FAF^* R_t + R_t \quad,\quad tr(FF^*)=1 \quad \text{and} \quad tr(FAF^*)=\gamma \tag{11}$$

Where γ is a positive constant. It can be proved that the largest desirable and feasible γ is [21]

$$\gamma=\sum_i \lambda_i(A)\lambda_i^*(FF^*) \tag{12}$$

After some calculations and by exploiting convex optimization and relaxation methods, F is equal to [21]

$$F=U_B\Lambda_B^{\frac{1}{2}}\Lambda_A^{-\frac{1}{2}}U_A^* \tag{13}$$

Where $B=FAF^*$ and U_A and Λ_A can be obtained from eigenvalue decomposition (EVD) of A and B (similar to [21]). We pass over the details as we are in the lack of space.

6. Simulation Results

In the simulations, it is assumed that the receiver perfectly has the channel knowledge and the transmitter only knows the mean and the correlation state of the channel. We assume that the total transmit power across all transmit antennas is unity. We use the ML decoding algorithm as an efficient and classic solution; however perfect space-time codes can be detected by sphere decoding [13] or other efficient and less complex methods such as MMSE-DFE-Fano decoders [18]. Fig. 2 demonstrates the BER performance of a system with Alamouti scheme in the presence of channel correlation. In Fig.3 and Fig.4 the correlation is calculated for $\rho=0.5$ and $\rho=0.8$. Fig. 3 shows the BER for the i.i.d case, the cases with correlation and with precoding using a codeword error matrix for the Golden code. In Fig. 4, the BER performance is depicted for a 4×4 perfect space-time block code. Clearly, Linear Precoding outperforms the correlated situation. This procedure can be used where this class of STBC is exploited for avoiding channel correlation.

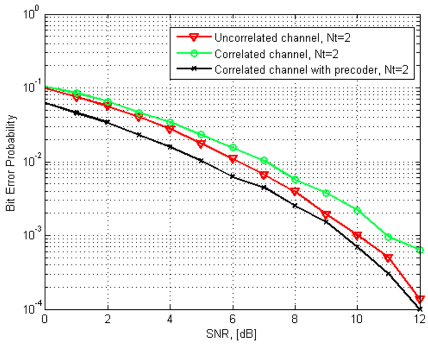

Fig. 2. Alamouti's scheme in the presence of channel correlation

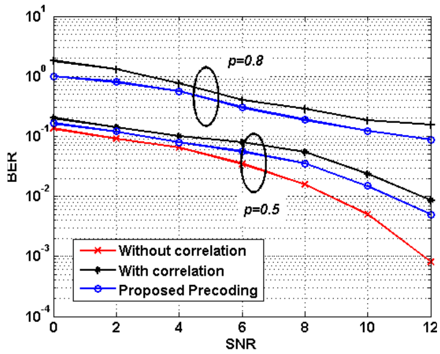

Fig. 3. Golden code(2×2) in the presence of channel correlation

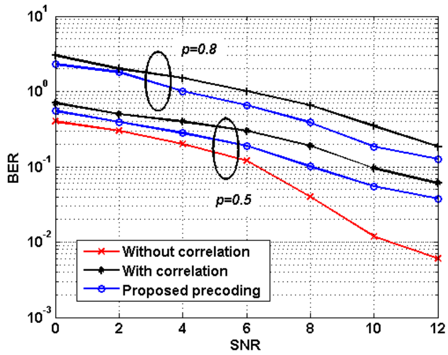

Fig. 4. 4×4 Perfect STBC scheme in the presence of channel correlation

7. Conclusion

In this paper we proposed a linear precoder for a perfect STBC MIMO system, in the correlation channel. MIMO antennas are normally correlated, due to the lack of spacing between them and this effect reduces the performance of the system. On the other hand the correlation phenomenon degrades the main characteristics of perfect codes; i.e. full diversity and full rate. In this contribution, we investigated the effect of channel correlation on diversity and coding gain of these codes. Afterwards, we designed a precoder for the PSTBC-based system to compensate the performance degradations due to correlation effects. Simulation results demonstrated that using the proposed precoder improve the performance of this system in correlated channel.

8. References

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