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A Fuzzy-AHP Approach to Prioritization of Critical Success Factors in Six Sigma Implementation: Evidence from Thailand

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Abstract. Six Sigma has been increasingly implemented and recognized as a powerful approach to achieve process improvements in both manufacturing and service industries. Although several studies have identified the success criteria and critical success factors for Six Sigma implementation but no specific studies have been carried out to discover which success factors are the most important than others. In order to promote the success of Six Sigma implementation, prioritization of critical success factors should be explored. This paper aims to present a fuzzy set theory based analytic hierarchy process (fuzzy-AHP) approach with the use of triangular fuzzy numbers for pairwise comparison scale for prioritizing success factors in Six Sigma implementation. The weights of critical success factors are determined by using the expertise, experience and knowledge of the nine experts including six managers, two Six Sigma Black-Belts and one Six Sigma Brown-Belt from three multinational companies located in Thailand, in the electronics industry. Finally, this paper concludes with contributions and suggestions for future research.

Keywords-Fuzzy-AHP; Six Sigma; Success factors; Prioritization

1. Introduction

Six Sigma implementation is to improve the quality of process outputs by identifying and removing the causes of defects (errors) and minimizing variability in manufacturing and business processes [1].

Six Sigma has not only been increasingly implemented in industry [2], but also increasingly recognized as a powerful approach to achieve process improvements in both manufacturing and service industries [3]. The successful implementation of Six Sigma within organizations can give them an edge over their competitors.

To promote the success of implementing Six Sigma, management team needs to address critical success factors (driving variables) more carefully for successful implementation of Six Sigma. However, there are some studies in the literature that investigate into critical success factors of Six Sigma implementation. Even though, all critical success factors are determined/ known, consequently organizations have to deal with a wide range of success factors. In fact, it is infeasible for organizations to devote their efforts to all critical success factors which are known.

To enhance the Six Sigma outcomes; the prioritization of critical success factors should be determined, and then some critical success factors with the highest prioritization should be emphasized during the Six Sigma implementation.

In order to incorporate the opinion, expertise, experience and knowledge of the experts, the fuzzy-AHP is applied to obtain more decisive judgments by prioritizing the critical success factors and weighting them in the presence of vagueness. The fuzzy-AHP approach is using the concepts of fuzzy set theory and hierarchical structure analysis to evaluate critical success factors in pairs, and quantify the relative importance of each factor to the successful implementation. The knowledge on the prioritizations of critical success factors of Six Sigma implementation will lead to better understanding of the operational and strategic management in the future.

2. Literature Review and Case Studies

In order to identify the success criteria and the factors for successful Six Sigma implementation, a large body of the relevant literature was reviewed. This led to the identification of three success criteria and five critical success factors is given below:

2.1. Success Criteria and Critical Success Factors of an Implementation of Six Sigma

The most critical success criteria of a Six Sigma implementation are in terms of (defect reductions resulting) yield improvements, improved customer satisfaction and higher net income. Findings from relevant literature, the success criteria for six sigma implementation in this study are as follows:

- Financial savings criteria [4], [5]: it is the results of defect reductions and yield improvements.
- Customer satisfaction criteria [4], [5]:
- Yield improvements: it is the results of defect reductions.

According to Brun's study [6] on "critical success factors" (CSF) of Six Sigma gathered from the 18 research papers, the five most frequently of CSF highlighted in these papers are adopted as CSF in this paper:

- Management involvement and commitment.
- Organizational infrastructure and culture.
- Education and training.
- Linking Six Sigma to business strategy.
- Linking Six Sigma to customer.

2.2. Case Studies

Our case studies are the manufacturers of computer-related products and a major player in the global market for several products, such as hard disk drives (HDDs), spindle motors for HDDs, semiconductors, and integrated circuit devices located in Thailand. These manufacturers have been dealing with Six Sigma projects such as cycle time reduction, minimizing their production/ operational costs, customer service improvement, and reducing response time for customer service excellence, etc. The following application of the fuzzy-AHP methodology to success factors prioritization for Six Sigma implementation will be presented by using the case studies.

2.3. The Panel of Experts

The panel of experts is formed based on their knowledge, experience, and skills of training and implementing Six Sigma projects. A balanced representation of experts with multiple perspectives should be incorporated in the decision-making process by selecting the experts from different organizations [7]. Our nine experts consisting of six managers, two Back-Belts and one brown-belt from three multinational companies located in Thailand, in the electronics industry. The experts have a mean experience in Six Sigma project/implementation of 7.3 years (SD = 3.3).

2.4. Suitability of Fuzzy-AHP for Success Factor Prioritization Problem in Six Sigma Implementation

The traditional AHP [8], [9] has been widely used across the industry in many applications such as for project selection [10], [11] allocating resources, and setting priorities [11], [12].

In this paper, we use a fuzzy-AHP approach for determining the weights for success factors of Six Sigma implementation, because, the weight determination problem primarily depends on subjective judgment or preference of the management and Six Sigma team.. In such a situation, it is difficult to incorporate preference scales in the analytical models. Therefore, using a crisp value for pairwise comparison is not suitable because it does not accurately represent the individual semantic cognition state of the decision makers [11]. Fuzzy logic [13] is a proven scientific technique that allows us to convert linguistic measures into crisp measure using membership functions [11].

2.5. The Fuzzy Set Theory

The fuzzy set theory [13] is designed to deal with the extraction of the primary possible outcome from a multiplicity of information vaguely and imprecisely. Fuzzy set theory treats vague data as possibility distributions in terms of set memberships.

Once determined and defined, the sets of memberships in possibility distributions can be effectively used in logical reasoning. Triangular fuzzy numbers are one of the major components. According to the definition of Laarhoven and Pedrycz [14], a triangular fuzzy number (TFN) should possess the following basic features.

The fuzzy number A on R to be a TFN if its membership function $\mu_A(x): R \to [0,1]$ is equal to

$$\mu_A(x) = \begin{cases} (x-l)/(m-l), & l \le x \le m, \\ (u-x)/(u-m), & m \le x \le u \\ 0, & otherwise \end{cases}$$

where l and u represent the lower and upper bounds of the fuzzy number A, respectively, and m is the median value. The TFN is denoted as $A = (l, m, u)$ and the following is the operational laws of two TFNs $A_1 = (l_1, m_1, u_1)$, $A_2 = (l_2, m_2, u_2)$, as shown [15], [16]:

Fuzzy number addition $\oplus$:

$$A_1 \oplus A_2 = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$$

Fuzzy number subtraction Θ:

$$A_1 \Theta A_2 = (l_1, m_1, u_1) \Theta (l_2, m_2, u_2) = (l_1 - l_2, m_1 - m_2, u_1 - u_2)$$

Fuzzy number multiplication $\otimes$:

$$A_1 \otimes A_2 = (l_1, m_1, u_1) \otimes (l_2, m_2, u_2) = (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2)$$

Fuzzy number division $\varnothing$:

$$A_1 \varnothing A_2 = (l_1, m_1, u_1) \varnothing (l_2, m_2, u_2) = (l_1 / u_2, m_1 / m_2, u_1 / l_2)$$

Fuzzy number reciprocal:

$$(\widetilde{A})^{-1} = (l, m, u)^{-1} \cong (1/u, 1/m, 1/l) \text{ for } l, m, u > 0$$

2.6. Fuzzy-AHP Method

With the AHP not being able to overcome the deficiency of the fuzziness during decision making, Laarhoven and Pedrycz [14] have evolved Saaty's AHP into the fuzzy-AHP, bringing the triangular fuzzy number of the fuzzy set theory directly into the pairwise comparison matrix of the AHP. The purpose is to solve vague problems, which occur during the analysis of criteria and judgment process. In this paper, Chang's extent analysis method [17], [18] is applied to the evaluation the critical success factors since the steps of this approach is similar to the traditional AHP and relatively easier than the other fuzzy-AHP approaches.

According to Chang's extent analysis method on fuzzy-AHP, triangular fuzzy numbers are used. Then each object is taken and extent analysis for each goal is performed respectively.

Triangular fuzzy numbers are used to represent subjective pair-wise comparisons of experts' judgments. The triangular fuzzy conversion scale used to convert such linguistic scales into fuzzy scales in the evaluation model is given in Table I.

TABLE I. MEMBERSHIP FUNCTION OF LINGUISTIC SCALE

Linguistic scales	Membership function
Equally important	(1, 1, 3)
Weakly important	(1, 3, 5)
Essentially important	(3, 5, 7)
Very strong important	(5, 7, 9)
Absolutely important	(7, 9, 9)

In the following, first the outlines of the extent analysis method on fuzzy AHP are given and then the method is applied to a prioritization problem. Let $X = \{x_1, x_2, \ldots, x_n\}$ be an object set, and $U = \{u_1, u_2, \ldots, u_m\}$ be a goal set. According to Chang's extent analysis [18], each object is taken and extent analysis for each goal is performed respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

$M_{g_i}^1, M_{g_i}^2, ..., M_{g_i}^m, \quad i = 1, 2, ..., n$

where all the $M_{g_i}^j \ (j = 1, 2, ..., m)$ are triangular fuzzy numbers.

The procedure of fuzzy-AHP can be given as in the following:

Step 1: Computing the value of fuzzy synthetic extent with respect to the i-th object, S_i .

Consider a triangular fuzzy comparison matrix expressed by

$$= \begin{bmatrix} M_{g1}^1 & M_{g1}^2 & \cdots & M_{g1}^m \\ M_{g2}^1 & M_{g2}^2 & \cdots & M_{g2}^m \\ \vdots & \vdots & \vdots & \vdots \\ M_{gn}^1 & M_{gn}^2 & \cdots & M_{gn}^m \end{bmatrix}$$

for $i = 1, 2, ..., n, \ j = 1, 2, ..., m$, if $i = j$ then $M_{g_i}^j = (1, 1, 1)$.

Then the value of fuzzy synthetic extent with respect to the i^{th} object is defined as

$$S_i = \sum_{j=1}^{m} M_{g_i}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j \right]^{-1} \tag{1}$$

Step 2: Computing the degree of possibility of $S_i \geq S_j$ by the following equation;

$$V(S_i \geq S_j) = height(S_j \cap S_i) = \begin{cases} 1, & if\ m_i \geq m_j \\ 0, & if\ l_j \geq u_i \\ \frac{l_j - u_i}{(m_i - u_i) - (m_j - l_j)}, & otherwise \end{cases}$$

where $S_i = (l_i, m_i, u_i)$ and $S_j = (l_j, m_j, u_j)$. To compare S_i and S_j , both the value of $V(S_i \geq S_j)$ and $V(S_j \geq S_i)$ must be calculated.

Step 3: Computing the minimum degree possibility: $Min\ V(S \geq S_1, S_2, ..., S_k)$

The degree of possibility of a convex fuzzy number to be greater than k convex fuzzy numbers S_i (for $i = 1, 2, ..., k$) can be defined as

$V(S \geq S_1, S_2, ..., S_k)$

$= V((S \geq S_1)\ and\ (S \geq S_2)\ and\ ...\ and\ (S \geq S_k))$

Therefore, $Min\ V(S \geq S_1, S_2, ..., S_k)$

$= Min\ V((S \geq S_1)\ and\ (S \geq S_2)\ and\ ...\ and\ (S \geq S_k))$

Assume that $w_i' = Min\ \ V(S_i \geq S_k)$ for $k = 1, 2, ..., n$; $k \neq i$, then the weight vector is defined as

$W' = (w_1', w_2', ..., w_n')^T$.

Step 4: Normalizing the weight vectors.

Via normalization, the normalized weight (or priority) vectors $W = (w_1, w_2, ..., w_n)^T$ are computed as follows:

$$w_i = \frac{w_i'}{\sum_{i=1}^{n} w_i'}$$

The priority vector W is a nonfuzzy (crisp) value.

3. Research Methodology

Via pairwise comparison, the pairwise comparison matrix called fuzzy judgment matrix is constructed. In this research, the triangular fuzzy number of the fuzzy set theory is brought directly into the pairwise comparison matrix of the AHP. The procedure of the fuzzy-AHP is described as follows:

Step 1: Defining the problem and objective

The objective of this study is the prioritization of critical success factors for Six Sigma implementation. This objective can be achieved by analyzing the effects of the critical success factors on the success criteria.

Step 2: Developing a hierarchy model

Fuzzy-AHP, an effective technique for analyzing a complex problem, can be applied for establishing weights in the hierarchical structure of environmental effects at each level. We establish a hierarchical structure for prioritization of critical success factors for Six Sigma implementation as shown in Figure 1. The hierarchical structure systematically accommodates the use of expert judgment.

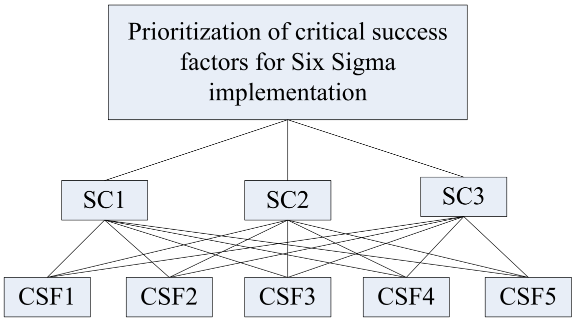

Figure 1. Hierarchical structure for prioritization of critical success factors for Six Sigma implementation

where,
SC1 = Financial savings
SC2 = Customer satisfaction
SC3 = Yield improvements
CSF1 = Management involvement and commitment
CSF2 = Organizational infrastructure and culture
CSF3 = Education and training
CSF4 = Linking Six Sigma to business strategy
CSF5 = Linking Six Sigma to customer

Figure 1 depicts a 3-level AHP model of prioritization of critical success factors for Six Sigma implementation, by presenting the structural relationship between the success criteria and the critical success factors. The first level of the model expresses the overall goal of this study, which is the prioritization of critical success factors for Six Sigma implementation. This goal can be achieved by analyzing the effects of the critical success factors on the success criteria. The second level presents the three of success criteria. The lowest level features the critical success factors.

Step 3: Establishing the fuzzy judgment matrix

We establish a fuzzy judgment matrix for each of the lower level elements, and then make the comparison of elements using the pair-wise comparison approach, the relative importance of the elements at the same level with respect to the element of their preceding level. Assign linguistic terms [11], [16] shown in Table I based on each expert's subjective judgments, to the pairwise comparisons by asking which one of two criteria (or critical success factors) is more important and how much more important is it with respect to the element of their preceding level.

Step 4: Aggregating opinions from several experts by using geometric mean

The informed judgments from a group of experts are aggregated through the geometric mean of individual experts' judgments. Let M_{ij}^{k} represent a subjective judgment of the k^{th} expert for the relative importance of two criteria (the ith criterion-the j^{th} criterion). The fuzzy geometric mean M_{ij} from m experts is as following equation

$$M_{ij} = (M_{ij}^{1} \otimes M_{ij}^{2} \otimes \cdots \otimes M_{ij}^{m})^{1/m}$$

4. Research Results

As results of fuzzy evaluation of criteria and success factors with respect to SC1 (there are other two fuzzy evaluation of success factors with respect to SC2 and SC3, but they are not presented in this paper of space limitations) which are the geometric mean values of the fuzzy judgment matrices are shown in Tables II and III.

TABLE II. FUZZY COMPARISON MATRIX OF THREE SUCCESS CRITERIA WITH RESPECT TO THE GOAL

	SC1	SC2	SC3
SC1	(1, 1, 1)	(0.42, 0.66, 0.96)	(0.50, 0.71, 1.58)
SC2	(1.04, 1.53, 2.37)	(1, 1, 1)	(1.23, 1.83, 2.53)
SC3	(0.63, 1.40, 2.01)	(0.39, 0.55, 0.81)	(1, 1, 1)

TABLE III. FUZZY COMPARISON MATRIX OF FIVE CRITICAL SUCCESS FACTORS WITH RESPECT TO SC1

	CSF1	CSF2	CSF3	CSF4	CSF5
CSF1	(1, 1, 1)	(0.89, 1.43, 3.18)	(0.81, 1.17, 2.63)	(0.91, 1.84, 3.43)	(0.40, 0.67, 1.31)
CSF2	(0.31, 0.70, 1.12)	(1, 1, 1)	(0.91, 1.24, 2.73)	(0.40, 0.53, 1.17)	(0.17, 0.22, 0.38)
CSF3	(0.38, 0.85, 1.24)	(0.37, 0.81, 1.10)	(1, 1, 1)	(0.41, 0.69, 1.35)	(0.43, 0.72, 1.43)
CSF4	(0.29, 0.54, 1.10)	(0.85, 1.89, 2.49)	(0.74, 1.44, 2.42)	(1, 1, 1)	(0.40, 0.60, 1.24)
CSF5	(0.76, 1.48, 2.49)	(2.63, 4.51, 5.96)	(0.70, 1.39, 2.35)	(0.81, 1.68, 2.49)	(1, 1, 1)

Taking the fuzzy judgment matrix of three criteria as an example, we calculate TFN values of three criteria by using the fuzzy evaluation values in Table II.

TFN values of criteria are as follows:

$$\begin{aligned} S_{1(SC1)} &= (1.92, 2.37, 3.54) \otimes (7.21, 9.68, 13.26)^{-1} \\ &= (1.92, 2.37, 3.54) \otimes (1/13.26, 1/9.68, 1/7.21) \\ &= (0.14, 0.24, 0.49) \end{aligned}$$

$$\begin{aligned} S_{2(SC2)} &= (3.27, 4.36, 5.90) \otimes (7.21, 9.68, 13.26)^{-1} \\ &= (3.27, 4.36, 5.90) \otimes (1/13.26, 1/9.68, 1/7.21) \\ &= (0.25, 0.45, 0.82) \end{aligned}$$

$$\begin{aligned} S_{3(SC3)} &= (2.02, 2.95, 3.82) \otimes (7.21, 9.68, 13.26)^{-1} \\ &= (2.02, 2.95, 3.82) \otimes (1/13.26, 1/9.68, 1/7.21) \\ &= (0.15, 0.30, 0.53) \end{aligned}$$

We calculate the degree of possibility of $S_i = (l_i, m_i, u_i) \geq S_j = (l_j, m_j, u_j)$ as follows:

$$V(S_{1(SC1)} \geq S_{2(SC2)}) = \frac{0.25 - 0.49}{(0.24 - 0.49) - (0.45 - 0.25)} = 0.53$$

$$V(S_{1(SC1)} \geq S_{3(SC3)}) = \frac{0.15 - 0.49}{(0.24 - 0.49) - (0.30 - 0.15)} = 0.85$$

$$V(S_{2(SC2)} \geq S_{1(SC1)}) = 1 \qquad (\textit{because of } 0.45 \geq 0.24)$$

$$V(S_{2(SC2)} \geq S_{3(SC3)}) = 1 \qquad (\textit{because of } 0.45 \geq 0.30)$$

$$V(S_{3(SC3)} \geq S_{1(SC1)}) = 1 \qquad (\textit{because of } 0.30 \geq 0.24)$$

$$V(S_{3(SC3)} \geq S_{2(SC2)}) = \frac{0.25 - 0.53}{(0.30 - 0.53) - (0.45 - 0.25)} = 0.65$$

We calculate the minimum degree possibility of $S_i \geq S_j$.

$$Min\, V(S_{1(SC1)} \geq S_{2(SC2)}, S_{3(SC3)}) = Min\,(0.53, 0.85) = 0.53$$

$$Min\, V(S_{2(SC2)} \geq S_{1(SC1)}, S_{3(SC3)}) = Min\,(1, 1) = 1$$

$$Min\, V(S_{3(SC3)} \geq S_{1(SC1)}, S_{2(SC2)}) = Min\,(1, 0.65) = 0.65$$

Then the weight vector obtained is as follows:

$W' = (0.53, 1, 0.65)^T$

Via normalization, the normalized weight vector obtained is as follows:

$W = (0.24, 0.46, 0.30)^T$

The calculations of local priority weights of the critical success factors with respect to the assessing criteria will not be given in this paper because they are similar to the calculation above. Finally, the overall priority weight of each success factor is calculated by multiplying its local priority weight with the 3 criteria relative weights. Table IV shows the results of prioritization for critical success factors.

TABLE IV. LOCAL AND OVERALL WEIGHT SCORE OF THE CRITICAL SUCCESS FACTORS AND THEIR PRIORITY RANKING

Critical Success Factor	**Local Weight Score**			**Overall Weight Score**	**Priority Ranking**
	SC1 (0.24)	*SC2 (0.46)*	*SC3 (0.30)*		
CSF1	0.23	0.24	0.22	0.232	2
CSF2	0.14	0.18	0.19	0.173	4
CSF3	0.14	0.11	0.13	0.123	5
CSF4	0.20	0.20	0.24	0.212	3
CSF5	0.29	0.27	0.22	0.260	1

Table IV shows the weights of critical success factors related to 3 criteria and also shows the overall weight and rank of each critical success factor. It can be concluded that from the experts' viewpoint through fuzzy-AHP approach, linking Six Sigma to customer plays the most important role in improving the implementation of Six Sigma. The experts consider that management involvement and commitment is the second most important one, while education and training is the least important factor. To achieve the successful of Six Sigma implementation, they believe that customer satisfaction criterion should be focused most on those.

5. Conclusion

This paper empirically studies the prioritization of the critical success factors in Six Sigma implementation, by using the fuzzy-AHP approach in order to incorporate the opinion, expertise, experience and knowledge of the experts. The factor prioritization is applied to the electronics industry in Thailand, and is presented to demonstrate efficiency of the proposed approach.

The prioritization of the critical success factors is very importance because it is infeasible to devote their efforts to all critical success factors. In promoting the success of implementing Six Sigma, management team and/or Six Sigma team need to devote their efforts to some critical success factors that have the highest prioritization such the linking Six Sigma to customer, and management involvement and commitment. Actually, in the beginning of implementing the Six Sigma, management team and/or Six Sigma team should focus their efforts on the critical success factor that has the highest prioritization and gradually attend to the rest of the factors which have running prioritization afterwards. Besides they need to emphasize the criterion of customer satisfaction as well.

Further studies may include sensibility analysis of the results of this study in order to determine the influence of these coefficients on the final result. The proposed approach of this study can be applied to different industries with more critical factors.

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