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Mirror Equivalent Turbo-Codes – Part II

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Abstract. this paper deals with the practical aspects of the mirror equivalence concept of convolutional codes and turbo-codes (TCs), already defined in a companion paper entitled first part. According to this new concept, the codes for which the code words are identical through transposition (reversal) are considered as equivalent (from the point of view of BER performance). In this paper we built the mirror equivalent TCs for two of the TCs used in the communication standards and we verify the mirror equivalence by simulations. The simulations performed prove that the pairs formed by the TCs from standards and their mirror equivalents have similar performance. The identification of some pairs of equivalent TCs simplifies the design of turbo-coding systems. There are large classes of sub-systems used in the design of a TC such as the component convolutional codes or the interleavers from where the best elements must be chosen. This is an exhaustive search. The idea of design based on exhaustive search of pairs of sub-systems is original and innovative. It is simplified if some of the TCs obtained are equivalent.

Keywords - convolutional codes; equivalence; generator matrix; permutation; turbo codes;

1. Introduction

The wide range of application of TCs highlights the importance of their design. If we take into account, for example, the TCs family that incorporates, as component codes, the memory 3 recursive systematic double binary convolutional codes (RSDBC) and the Quadratic Polynomial Permutations (QPP) interleavers, [1], of length N=752, we observe that this family counts over a million components[1]. An exhaustive search of the best pairs: component code-interleaver; over such a large set is very laborious. In the same time this is an innovative idea because until now the optimization of each sub-system composing a TC was realized independently.

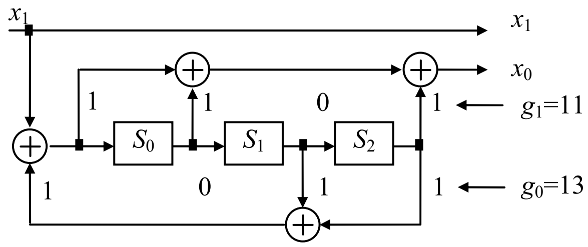
This complexity amplifies exponentially with the increase of the memory or with the inclusion of new families of TCs [2]. The idea of eliminating some competitors, knowing that they are equivalent as performance with others elements already investigated becomes attractive and useful. We defined the equivalence of such TCs and we showed the theoretical conditions in which two (turbo) codes are mirror equivalent in [3]. The present paper highlights the practical advantages of the new concept of mirror equivalence. More precisely, the simulations reported in this paper confirm the similarity of the performances for pairs of mirror equivalent TCs. The paper structure is the following. The subject of the second section is a short presentation of two turbo-codes used in present standards of communications. A brief presentation of the new concept of mirror equivalence and of the necessary conditions for two TCs to be mirror equivalent is made in the third section. Some TCs classes including the TCs presented in the second section are detailed. The forth section compares the performances of standard TCs, obtained through simulations, with those of their mirror equivalent TCs. In the fifth section are drawn a few conclusions.

2. Standard Turbo-Codes

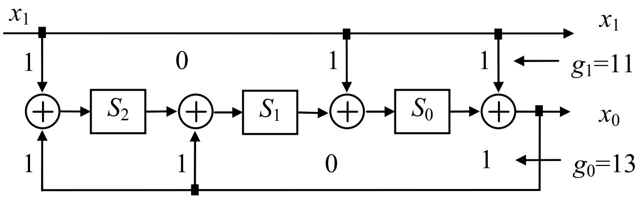
[1] It is an estimation of the total number of TCs, without taking into account their performance;

A. The TC used by the standard 3GPP

The scheme of the component encoder of the TC used in the standard “Technical Specification Group Radio Access Network, E-UTRA” (3GPP) [4], in controller canonical form (also taken from [4]) is presented in Fig.1a.



a) controller canonical form



b) observer canonical form

Fig.1 The scheme of the component code of the TC specified in the standard 3GPP-RAN, having the generator matrix G_{3gpp}=[11 13].

The implementation of the same encoder in the observer form is presented in Fig. 1b. The generator matrix (see [3] or [5]) and the input-output relation of the system have the following expressions:

$$G_{3gpp}(D) = [1 \quad g_1(D)/g_0(D)], \tag{1}$$

$$x_0(D) = \frac{g_1(D)}{g_0(D)} \cdot x_1(D), \tag{2}$$

where $g_0(D) = 1 + D^2 + D^3$, $g_1(D) = 1 + D + D^3$, and $x_i(D) = \sum_{j=0}^{N-1} x_{i,j} \cdot D^j$, $0 \le i \le 1$, represent the D transforms of the information sequences and of the redundancy sequences respectively. So, the coded word is: $y(D) = x(D) \cdot G_{3gpp}(D) = [x_1(D) \quad x_0(D)]$. For the sake of simplicity we will use from now on the denotation (in decimal): G_{3gpp}=[11 13].

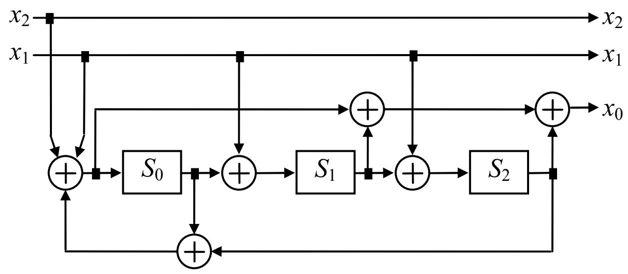
In the standard it is provided for an Un-interleaved Dual Termination (UDT) trellis’ closing type and a QPP interleaver with the interleaving function of the form:

$$\pi_{QPP}(i) = (f_1 \cdot i + f_2 \cdot i^2) \bmod N,\ 0 \le i < N, \tag{3}$$

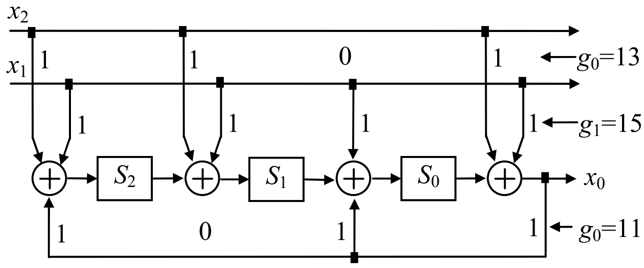
where N represents the interleaving length. For N the standard provides for 188 values between 40 and 6144.

B. The TC used in the standard DVB RCS

The controller and observer canonical forms for the RSDBC encoder used in the standard Digital Video Broadcasting - Return Channel via Satellite (DVB RCS) [6] are presented in Fig. 2.



a) controller canonical form



b) observer canonical form

Fig.2 The scheme of the component code of the TC specified in the standard DVB-RCS, having the generator matrix G_{DVB}=[13 15 11].

The generator matrix and the input-output relation of the system have the expressions:

$$G_{DVB}(D)=\begin{bmatrix}1 & 1 & g_2(D)/g_0(D)\\ 0 & 0 & g_1(D)/g_0(D)\end{bmatrix}, \tag{4}$$

$$x_0(D)=\frac{g_2(D)}{g_0(D)}\cdot x_2(D)+\frac{g_1(D)}{g_0(D)}\cdot x_1(D), \tag{5}$$

where $g_0(D)=1+D+D^3$, $g_1(D)=1+D+D^2+D^3$ and $g_2(D)=1+D^2+D^3$, $x_1(D)$ and $x_2(D)$ are the polynomials of the input sequences (their D transforms) and $x_0(D)$ is the redundancy. Consensually with the previous paragraph we will use from now on the denotation (in decimal): G_{DVB}=[13 15 11], and the code word has the form $y(D)=x(D)\cdot G_{DVB}(D)=[x_2(D)\quad x_1(D)\quad x_0(D)]$.

There are two levels of interleaving; intra- and inter-symbol. The intra-symbol interleaving supposes the inversion of bits' positions in the symbols with even index. The inter-symbol interleaving is ensured by an ARP (Almost Regular Permutation) interleaver, [7], having the function:

$$\pi_{ARP}(j) = (P_0 \cdot j + P) \bmod N,\ 0\le j<N, \tag{6}$$

where $P = 0$ if j mod 4 = 0, $P = N / 2 + P_1$ if j mod 4 = 1, $P = P_2$ if j mod 4 = 2 and $P = N / 2 + P_3$ if j mod 4 = 3. Parameters P_0, P_1, P_2, and P_3 depend on N. The data block dimension (with the form $2\times N$) can take the values: 12, 16, 53, 55, 57, 106, 108, 110, 188, 212, 214 and 216 bytes. It is used a circular closing (tail biting), [8].

3. Mirror Equivalence

C. Mirror equivalent convolutional codes

We have proposed in [3] the following definition of the mirror equivalence.

Definition 1. Two convolutional codes, denoted as $\mathcal{C}_{G,N}$, and $\mathcal{C}_{Gd,N}$, generated by the encoders whose generator matrices are G şi G_d, having words of length N, are *mirror equivalent*, if the generated code words are the mirror images:

$\forall\, y\in \mathcal{C}_{G,N}$, then $y_d=\mathfrak{M}(y)\in \mathcal{C}_{Gd,N}$ and

$\forall\, y_d\in \mathcal{C}_{Gd,N}$, then $y=\mathfrak{M}(y_d)\in \mathcal{C}_{G,N}$,

where: $\mathfrak{M}(y(D)) = y(D^{-1})\cdot D^{N-1}$, denotes the mirroring operation, applicable for the polynomial with finite dimension N: $y(D)=\sum_{j=0}^{N-1} y_j\cdot D^j$.

The condition that two codes to be mirror equivalent is given by the following theorem [3].

Theorem 1. The codes $\mathcal{C}_{G,N}$ and $\mathcal{C}_{Gd,N}$ generated by the encoders having the matrices $G=[g_k \ldots g_{i+1}\ \mathfrak{M}(g_0)\ g_{i-1} \ldots g_0]$ and $G_d=[\mathfrak{M}(g_k) \ldots \mathfrak{M}(g_{i+1})\ g_0\ \mathfrak{M}(g_{i-1}) \ldots \mathfrak{M}(g_0)] = \mathfrak{M}(G)$ are mirror equivalent, with $1\le i\le k$.

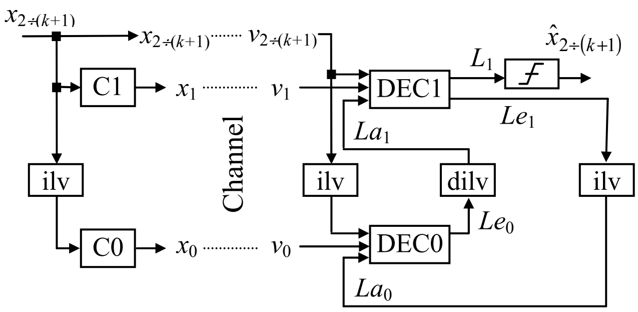


Fig. 3 The TC's general scheme

In order to exist a mirror equivalent, the generator matrix from the statement of theorem 1 should have the following form G=[g_k ... g_{i+1} $\mathfrak{M}(g_0)$ g_{i-1} ... g_0]. So, among the generator polynomials corresponding to those k inputs there should be found the mirror image polynomial of the feedback polynomial g_0. This condition is satisfied by both generator matrices, G_{3gpp}=[11 13] and G_{DVB}=[13 15 11]. The dual mirror equivalent codes are generated by the matrices G_{3gppd}=[13 11] and G_{DVBd}=[11 15 13].

D. Mirror equivalent (convolutional) turbo-codes

With the denotations in Fig. 3, we will use the following definition of TC (see [3]).

Definition 2. The TC, denoted by $\mathfrak{TC}_{G,N,\pi}$, generated by the component encoders which implement the generator matrix $G=[g_k \ \ldots \ g_1 \ g_0]$ and the interleaver defined by the permutation π:$\mathcal{K}\times\mathcal{T}\rightarrow\mathcal{K}\times\mathcal{T}$, where $\mathcal{K}$={1,2, ..., k} and $\mathcal{T}$={0,1,2, ..., N-1}, is constituted from the multitude of words of form $y_{TC}=[x_{k+1} \ \ldots \ x_2 \ x_1 \ x_0]$, where x_1 and x_0 are redundant sequences generated by the two encoders C1 and C0.

The following theorem was formulated in [3] on the basis of this definition.

Theorem 2. Two TCs, denoted by $\mathfrak{TC}_{G,N,\pi}$ and $\mathfrak{TC}_{Gd,N,\pi d}$, are mirror equivalent if the component codes, $\mathcal{C}_{G,N}$ and $\mathcal{C}_{Gd,N}$, are mirror equivalent and the corresponding interleaver functions satisfy the condition:

$$\pi_d(\mathfrak{M}(u))=\mathfrak{M}(\pi(u)), \tag{7}$$

where u is a certain input sequence for the encoder defined by the matrix G.

We will customize (7) for the case of the TCs defined in section 2. Because there is no inter-symbol interleaving for the single-binary TC, in the case of 3GPP TC, denoted in the following as $\mathfrak{TC}_{3gpp}$, the condition (7) is reduced to the relation:

$$\pi_d(N-i-1)=N-\pi(i)-1, \ 0\leq i<N. \tag{8}$$

TABLE I. GENERATOR POLYNOMIALS UNDER MIRROR EQUIVALENCE

m	g_0	$\mathfrak{M}(g_0)$	g_x	N_{cu}	N_{cb}
0	-	-	1	-	-
1	-	-	3	-	-
2	-	-	5, 7	-	-
3	11	13	9, 15	1	(16-3)×2=26
4	19 23	25 29	17, 21, 27, 31	2	3×(32-3)×2=174
5	35 37 39 43 47 55	49 41 57 53 61 59	33, 45, 51, 63	6	9×(64-3)×2=1098

in which f_{0d}=(f_1-f_2-1)mod$N$, f_{1d}=(f_1-2·f_2)mod$N$ and f_{2d}=(N-f_2)mod$N$. It results that for the dual mirror equivalent TC of $\mathfrak{TC}_{3gpp}$, denoted as $\mathfrak{TC}_{3gppd}$, we have the following interleaver function:

$$\pi_{QPPd}(i) = N-\pi_{QPP}(N-i-1) -1= (f_{0d} + f_{1d}\cdot i + f_{2d}\cdot i^2) \bmod N, \ 0\leq i<N, \tag{9}$$

In the case of the duo-binary TC used in the standard DVB-RCS, denoted as $\mathfrak{TC}_{DVB}$, it appears in addition the inter-symbol interleaving. Because this does not modify the permutation of the symbols, the latter one generates a function of dual-interleaving of the form:

$$\pi_{ARPd}(i) = N\text{-}\pi_{ARP}(N\text{-}i\text{-}1)\ \text{-}1= (P_0 \cdot i + P_d) \bmod N,\ 0 \leq i < N, \qquad (10)$$

where P_d=(P_0-P-1)mod$N$. This interleaver together with an interchange of odd symbols bits (fact imposed by the condition (7)) furnishes the interleaving rule of the dual mirror equivalent TC, denoted by $\mathfrak{TC}_{DVBd}$. As we previously specified, the component code of the $\mathfrak{TC}_{3gppd}$ is generated by the matrix G_{3gppd}, and for the $\mathfrak{TC}_{DVBd}$ we have the generator matrix G_{DVBd}.

For a single-binary TC to have a mirror equivalent it is necessary that its generator matrix to be of the form G=[$\mathfrak{M}(g_0)$ g_0]. The dual TC has the generator matrix G_d=[g_0 $\mathfrak{M}(g_0)$]. The interleaver functions satisfy a general condition of the form (7). Particularizations of type (9) or (10) become applicable if QPP or ARP interleavers are used. For the duo-binary case, the generator matrix G may have the form [$\mathfrak{M}(g_0)$ g_1 g_0], respectively [g_1 $\mathfrak{M}(g_0)$ g_0]. Table 1 shows the polynomials of a maximum degree of 5 (memory = m) whose mirror images polynomials are different but have the same degree. In addition, there are presented the g_x polynomials for which $\mathfrak{M}(g_x) = g_x$ as well. These cannot be used as g_0 polynomials because it would result non efficient generator matrices (having two identical columns). There are also primitive polynomials marked in red. In the last two columns of table 1, we have indicated the number of recursive and systematic convolutional code pairs, which can

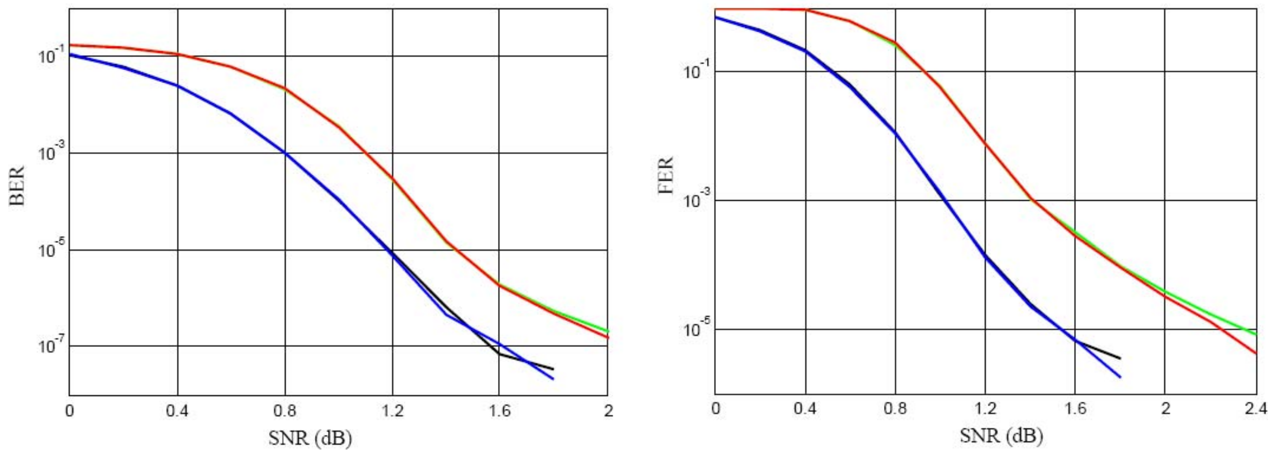


Fig. 4 BER/FER versus SNR performances for the TCs: $\mathfrak{TC}_{3gpp}$ (black), $\mathfrak{TC}_{3gppd}$ (blue), $\mathfrak{TC}_{DVB}$ (green) and $\mathfrak{TC}_{DVBd}$ (red) in the IDT case.

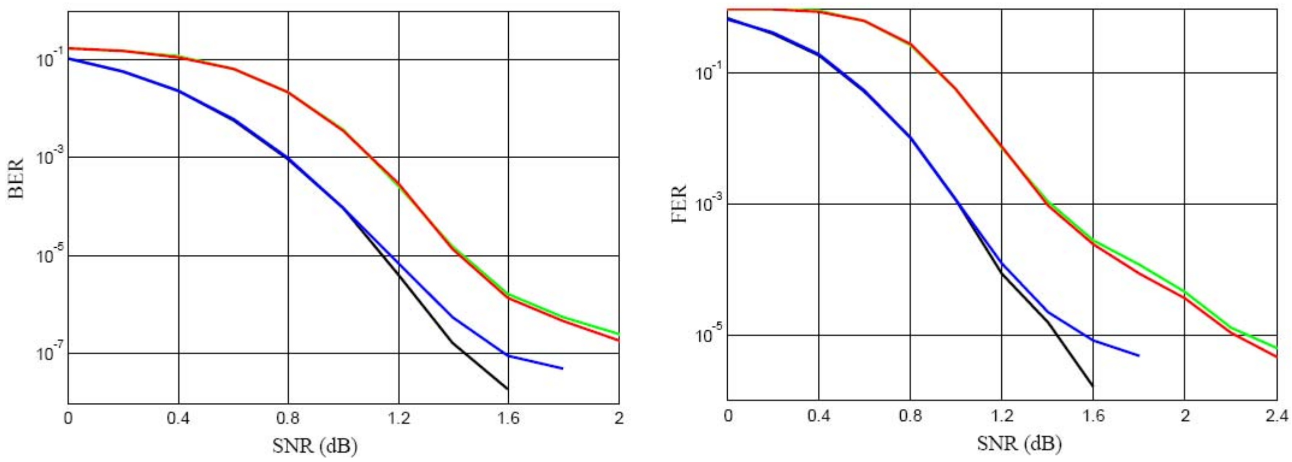


Fig. 5 BER/FER versus SNR performances for the TCs: $\mathfrak{TC}_{3gpp}$ (black), $\mathfrak{TC}_{3gppd}$ (blue), $\mathfrak{TC}_{DVB}$ (green) and $\mathfrak{TC}_{DVBd}$ (red) in the tail biting case.

be formed, for the single-binary (N_{cu}) and duo-binary (N_{cb}) cases.

4. Mirror equivalent TC performance

In this section we present a comparison of the performances of $\mathfrak{TC}_{3gpp}$ and $\mathfrak{TC}_{3gppd}$ and of $\mathfrak{TC}_{DVB}$ and $\mathfrak{TC}_{DVBd}$, respectively, customized for the interleaving length N=752.

For this length, in the 3GPP-RAN standard there are specified the QPP interleaver parameters, f_1=23 and f_2=94. The necessary interleaver for the $\mathfrak{TC}_{3gppd}$ is of the form (9) where f_{0d}= 680, f_{1d}= 587 and f_{2d}= 658. For the $\mathfrak{TC}_{DVB}$, the parameters of the ARP interleaver (6) which is recommended for the length N=752 (1504 bits) are P_0=19, P_1=376, P_2=224 and P_3 =600. The interleaver for the $\mathfrak{TC}_{DVBd}$ is defined by (10).

The performances of the two pairs of mirror equivalent TCs are presented in the diagrams from Fig.4 and Fig.5. Different trellises' closings are used in these figures, explaining the difference between the diagrams. The curves in Fig. 4 were obtained using an IDT closing (which was

considered in the definition of mirror equivalence [3]**)**. The simulations were stopped when 1000 erroneous blocs were obtained for all the points in diagrams with the exception of the points with $FER \leq 10^{-5}$, for which the simulations were stopped when 100 erroneous blocks were obtained. The coincidence almost complete of the curves proves that the TCs identified as mirror equivalent have identical performances. Because the mirror equivalence theory [3] is based on the IDT closing, the following question arrives. "What is happening in the case when other closings are used?" For the tail biting closing, the answer is given in Fig. 5. In the case of duo-binary TCs, the equivalence is kept, having similar performances for the equivalent TCs. The difference in performance for the single-binary case is surprisingly.

The TC recommended by the standard has better performance for the tail biting closing than its mirror equivalent. To identify the cause of this difference we have repeated the simulations in the binary case making a reversal of interleavers (we have associated the code from the standard and the dual interleaver and the dual code with the interleaver recommended by the standard). The results obtained (not presented here) are similar with the results already presented. This time the pair with better performance is composed by the dual code and the interleaver from the standard. This fact suggests the idea that the two codes remain equivalent if the same interleaver is used.

The results in Fig. 5 suggest some questions: „Can we find interleavers for which the mirror equivalence can be extended to the case of tail biting closing?", „Does the ARP interleaver satisfy such conditions?". The responses could represent results of future research.

5. Conclusions

In this paper, we have given supplementary details about the mirror equivalence of correcting codes and TCs, defined in [3]. This type of equivalence is original and simplifies the TCs design procedure based on the exhaustive search of the best pairs of constituent convolutional code – interleaver. The TCs design procedure already mentioned is innovative.

We derived and designed the mirror equivalent TCs for the TCs used in the standards 3GPP-RAN and DVB-RCS respectively. For these pairs of mirror equivalent TCs we have comparatively presented the BER/FER versus SNR performance. The identity of those curves confirms the equivalence of the pairs of TCs investigated and the correctness of our designs. The difference in performance introduced by the use of different closings (IDT and tail biting) highlights the necessity to use the IDT closing for the mirror equivalence, as was considered in [3]. The concept of mirror equivalence can be used for any block code using the IDT closing.

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