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Spatial Correlation Effects on Channel Estimation of UCA-MIMO Receivers

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Abstract – This paper presents investigations into the effect of spatial correlation on channel estimation and capacity of a multiple input multiple output that employ Uniform Circular Array at the receiver end (UCA-MIMO) wireless communication system. Least square (LS), scaled least square (SLS) and minimum mean square error (MMSE) methods and relaxed MMSE (RMMSE) are considered for estimating channel properties of a MIMO system using training sequences. Performance of MMSE estimation method under different spatial correlation conditions is studied. The effect of varying the SNR on the Channel State Information (CSI) error and capacity for UCA-MIMO systems is also presented by performing system simulation that includes an accurate and realistic channel model.

1. Introduction

Most MIMO systems are based on perfect channel knowledge being available at the receiver. However, perfect channel knowledge is never known a priori. In practice, the channel has to be estimated to acquire the channel state Information (CSI) at the receiver and at the transmitter in some cases. Therefore, accurate and efficient channel estimation plays a key role in MIMO communication systems. The MIMO channel estimation can be classified into two methods. Firstly, the data aided, pilot based method that is based on training symbols with a priori known at the receiver [1]. The second is a non-data aided, blind based, method that relies only on the received symbols [2]. In these blind techniques, CSI is attained by exploiting statistical information and/or transmitted symbols properties (like finite alphabet, constant modulus, etc.). However, compared with training, blind channel estimation generally requires a long data record and higher complexity. Therefore, this work focuses on the data aided based channel estimation method performance under different channel conditions.

In [1], a number of training based methods have been studied including the least squares (LS), the scaled least squares (SLS), the linear minimum mean square error (MMSE), and Relaxed minimum mean square error (RMMSE). The optimal training sequence designs are introduced for MIMO systems in [3]. In [4], it has been demonstrated that the presence of spatial correlation can help to achieve a better quality CSI. Most of the previous work was performed for ULA geometry of MIMO array. In [5], the impact of channel spatial correlation on the channel estimation error is evaluated when UCA antenna is employed at the receiver side. In this paper, LS, SLS, MMSE and RMMSE training-based channel estimation methods are implemented and studied by applying an accurate channel model for MIMO system [6]. The performances of channel estimators are investigated when applying optimum training sequences and orthogonal sequences. Also, channel estimation errors and capacity for UCA-MIMO systems at various AOA and AS values are presented. This paper is organized as follows; the spatially correlated MIMO channel model is presented in Section 2. In section 3, LS, SLS, LMMSE, and RMMSE channel estimation methods are studied and compared under the condition of spatial correlation. Section 4 presents the numerical results. Finally, conclusions are derived in section 5.

2. Fading Channel Model

A correlated fading channel is considered for MIMO system with M_t transmit antennas and M_r receive antennas [6]. The received signal in the training mode is expressed as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{V} \tag{1}$$

where **Y** is the $M_t \times N$ complex matrix representing the received signals, **X** is the $M_t \times N$ complex training matrix, which includes training pilot sequences; **V** is the $M_r \times N$ complex zero mean white noise matrix; N is the length of the transmitted training signal; **H** is the $M_r \times M_t$ complex channel matrix at one instance of time can be modeled as a fixed (constant, LOS) matrix and a Rayleigh (variable, NLOS) matrix.

$$\mathbf{H} = \sqrt{\frac{K}{1+K}}\mathbf{H}_f + \sqrt{\frac{K}{1+K}}\mathbf{R}_r^{1/2}\mathbf{H}_w\mathbf{R}_t^{1/2} \tag{2}$$

where $\mathbf{H}_f$ represents the fixed LOS channel matrix and $\mathbf{H}_w$ is zero mean and unit variance complex Gaussian random variables that presents the coefficients of the variable NLOS matrix, K is the Rician K-factor. $\mathbf{R}_r$ and $\mathbf{R}_t$ are the $M_r{\times}M_r$ and $M_t{\times}M_t$ receiver and transmitter spatial correlation matrices respectevly and are obtained as in [6] for both ULA-MIMO and UCA-MIMO configurations.

3. Channel Estimation under Spatial Correlation Conditions

3.1. LS Channel Estimator

Knowing Q and Y, the traditional least squares (LS) estimate for the channel matrix is given in [1]

$$\widehat{\mathbf{H}}_{\mathrm{LS}} = \mathbf{YQ}^{\dagger} \tag{3}$$

where $\mathbf{Q}^{\dagger} = \mathbf{Q}^{\mathrm{H}} (\mathbf{Q}^{\mathrm{H}}\mathbf{Q})^{-1}$ is the Moore-Penrose Pseudo-inverse of **Q** and $[.]^{\mathrm{H}}$ denotes the Hermitian transpose. As can be seen, the estimate doesn't require any knowledge about the channel parameters. The minimum MSE of LS estimator is

$$\sigma_{\mathrm{LS}} = \frac{\sigma_n^2 M_t^2 M_r}{\rho_x} \tag{4}$$

where ρ_x/σ_n^2 is the transmitted power to noise ratio (TPNR) in training mode. The optimal performance of the LS is influenced by the square of number of antenna elements at the transmitter and by the number of antenna elements at the receiver. However, the channel matrix has no effect on the MSE.

3.2. SLS Channel Estimator

The SLS channel estimated matrix is

$$\widehat{\mathbf{H}}_{\mathrm{SLS}} = \gamma_o \widehat{\mathbf{H}}_{\mathrm{LS}} = \frac{\mathrm{tr}\{\mathbf{R}_H\}}{\sigma_n^2 M_r \mathrm{tr}\{(\mathbf{QQ}^{\mathrm{H}})^{-1}\} + \mathrm{tr}\{\mathbf{R}_H\}}\mathbf{YQ}^{\dagger} \tag{5}$$

Here, σ_n^2 is the noise power; $\mathbf{R}_H$ is the channel correlation matrix defined as $\mathbf{R}_H = \mathrm{E}\{\mathbf{H}^{\mathrm{H}}\ \mathbf{H}\}$ and tr{.} implies the trace operation. In practice, $\mathbf{R}_H$ can be obtained using the channel matrix estimated by the LS method, in this case the resulting estimator is referred to the LS-SLS. Accordingly, under the optimal training the MSE is

$$\sigma_{\mathrm{SLS}} = [\mathrm{tr}\{\mathbf{R}_H\}^{-1} + (\sigma_{\mathrm{LS}})^{-1}]^{-1} \tag{6}$$

$\mathrm{tr}\{\mathbf{R}_H\}^{-1} = \sum_i^n \lambda_i$, λi the i-th eignvalue of the channel correlation $\mathbf{R}_H$.

3.3. Mmse Channel Estimator

The estimated channel matrix of MMSE method is

$$\widehat{\mathbf{H}}_{\mathrm{MMSE}} = \mathbf{Y}\left(\mathbf{Q}^{\mathrm{H}}\mathbf{R}_H\mathbf{Q} + \sigma_n^2 M_r \mathbf{I}\right)^{-1}\mathbf{Q}^{\mathrm{H}}\mathbf{R}_H \tag{7}$$

The MSE of the MMSE can be expressed as

$$\sigma_{\mathrm{MMSE}} = \mathrm{tr}\{(\mathbf{R}_{\mathrm{H}}{}^{-1} + \frac{\mathbf{QQ}^{\mathrm{H}}}{\sigma_n^2 M_r})^{-1}\} = \mathrm{tr}\{(\mathbf{\Lambda}^{-1} + \widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^{\mathrm{H}})^{-1}\} \tag{8}$$

where $\mathbf{R}_H = \mathbf{U\Lambda U}^{\mathrm{H}}$, **U** is the unitary eigenvector matrix of $\mathbf{R}_H$ and $\mathbf{\Lambda}$ is the diagonal matrix with eigenvalues of $\mathbf{R}_H$.

$$\widetilde{\mathbf{Q}} = \frac{\mathbf{U}^{\mathrm{H}}\mathbf{Q}}{\sqrt{\sigma_n^2 M_r}} \tag{9}$$

The optimal training matrix of the LMMSE can be derived by using the Lagrange multiplier method that yield to optimum training matrix as

$$\mathbf{Q} = \sqrt{\sigma_n^2 M_r}\mathbf{U}\sqrt{(\mu_o \mathbf{I} - \boldsymbol{\Lambda}^{-1})^{+}} \tag{10}$$

where (x)+ is the $max(x,0)$ and the constant $\mu_o = 1/\sqrt{\mu}$ has to be adjusted to satisfy the transmitted power constraint

3.4. RMMSE Channel Estimator

The MMSE channel estimator (7) assumes the perfect knowledge of the matrix RH. However, in practice this assumption is unrealistic. Thus, the LMMSE estimator is relaxed and simplified by replacing RH with the matrix δI, where the parameter δ has to be adjusted to minimize the MSE. Hence, (7) can be written as

$$\widehat{\mathbf{H}}_{\mathbf{RMMSE}} = \mathbf{Y}\left[\mathbf{Q}^{\mathbf{H}}\mathbf{Q} + \frac{\sigma_n^2 M_t M_r}{\mathbf{tr}\{\mathbf{R}_H\}}\mathbf{I}\right]^{-1}\mathbf{Q}^{\mathbf{H}} \tag{11}$$

The RMMSE estimation error for an orthogonal training is given by [1]

$$\sigma_{\mathrm{RMMSE}} = \left[\mathrm{tr}\{\mathbf{R_H}\}^{-1} + \left(\frac{\rho_x}{\sigma_n^2 M_t^2 M_r}\right)\right]^{-1} \tag{12}$$

4. Numerical Results

In this section, we study the channel Estimation Error for two systems with ULA-MIMO and UCA-MIMO receivers. In both systems, the transmitter MIMO antenna is assumed to be ULA with inter-elements distance (d_t). ULA-MIMO receive antenna has a uniform inter-elements spacing (d_r) and placed vertically such that AoA= 0 is broadside case. UCA-MIMO receive antenna has radius (R_r). The numerical studies are performed for MIMO systems with channel model simulation as in [6] where WLAN uplink scenario is modeled with transmitter at the mobile unite (MU) and receiver at the base station (BS). The channel is modeled as multi-clusers scattering environment which means that the signal will arrive at the BS from multiple Angles of arrival (AoA) each with angle spread (AS) that is a measure of the angle displacement due to the non-LOS propagation. The LS, SLS, MMSE, and RMMSE channel estimators are implemented at the receiver. The following parameters are considered, $N=M_t$, truncated Laplacian Powe Azmiuth Spectrum (PAS) distribution, 10000 channel realizations, and the MSE is normalized by $M_t \times M_r$. Fig. 1 demonstrates the normalized MSEs of the LS, SLS, MMSE, and RMMSE channel estimators with orthogonal training versus SNR, for 4×4 ULA–UCA MIMO system. As seen, the LS estimator has the worst performance, while the MMSE has the best performance among all techniques. Meanwhile, it requires more a prior knowledge about the channel than other methods. SLS and RMMSE estimators are identical and they necessitate less prior knowledge of the channel than the MMSE estimator. Therefore, the selection of the channel estimator requires a tradeoff between the given performance and the available channel knowledge. For the rest of the paper, MMSE is considered to study the effect of spatial correlation on estimation error for both ULA and UCA geometries. Fig. 2 shows the normalized MSEs of the MMSE estimator versus SNR with both orthogonal and optimal training for M_r=4, and M_t =2, 4. It can be observed that the performance of the optimal training is better than the orthogonal training especially at low SNRs. It is also noticed that when M_t is small and SNR is high the orthogonal training is nearly optimal. Fig. 3 illustrates the normalized MSE versus AoA of MMSE channel estimator with AS=20 at various SNR values of 0 dB, 10 dB, and 20 dB for 4 elements ULA and UCA MIMO antennas utilized at the receiver end. It can be seen that at high SNR the MSE is less and the geometry has a no pronounced effect on improving the channel estimation error. The presented result reveals that for low SNR the estimation error has more variations in ULA-MIMO geometry due to the variable fading correlations values at different AOAs. Also, it can be seen that the performances of both geometries are identical near broadside angles {AOA=0^o and 180^o} where the correlation is minimum. ULA outperforms the UCA at endfire angle {AOA=90^o} where the correlation has its maximum value. So, the minimum channel estimation error can be achieved by employing ULA if the receiver is expecting to have the signals arriving at endfire angles. In Fig. 4 and Fig. 5 present the 3-D graphs showing the relationship between the normalized MSE of MMSE channel estimator versus AS and AOA for ULA-MIMO and UCA-MIMO receivers respectively. Here the SNR is assumed to be 10 dB. The figures show that as AS increases (spatial correlation decreases), the performance of channel estimation gets worse. In addition, for higher AS, the MSE value varies when AOA changes. In contrary, for small AS the MSE becomes independent of the value of AOA for both geometries. In Fig. 4 for ULA-MIMO, the best performance at endfire angle AoA=90^o and at low AS. On the other hand, In Fig. 5 for UCA-MIMO, the minimum MSE can be attained with AOA = 45^o and 135^o due to the fact that, two elements are directly behind and parallel to the other two elements (highest correlation), this can be noticed particularly at high AS. From the presented results, it can be concluded that the existence of spatial correlation improves channel estimator performance for UCA-MIMO as well as it does for ULA-MIMO receivers. Fig. 6 shows the Ergodic capacity of UCA-MIMO systems with MMSE channel estimator versus SNR at different AS values. At high SNR values, as AS decreases and spatial correlation increases, the capacity decreases However, it is noticed that at low SNR ≤ 6dB, as angle spread decreases (spatial correlation has more effect) channel capacity increases for MMSE-UCA-MIMO systems. This disobeys the conventional knowledge that spatial correlation reduces the channel capacity.

5. Conclusion

The data-aided (training or pilot based) channel estimation method has been studied. The LS, SLS, LMMSE, and RMMSE channel estimators have been demonstrated. Orthogonal and optimal training symbols performances are presented for MMSE. The results have been confirmed that MMSE method offers best performance over the other methods. This is for the reason that of utilizing the channel correlation that reduces the channel estimation error in the previous- methods, while the LS method does not consider the channel properties. However, it requires more a *prior* knowledge about the channel than the other methods. The SLS and RMMSE necessitate less *prior* knowledge about the channel than the MMSE estimator. In this paper, the impact of channel spatial correlation on the accuracy of MIMO channel estimation error has been investigated. The undertaken analysis has revealed that the strongly correlated channel can improve the channel estimation at low SNR for the considered UCA-MIMO systems. However, at high SNR the channel spatial correlation has less effect pronounced on the accuracy of the channel estimation. In addition the results demonstrate that the performance of the channel estimator in ULA-MIMO system has variation when having spatial correlations by varying AOA or AS. However, even with this variation the MSE of MMSE channel estimator for ULA-MIMO systems has in general less values than that for UCA-MIMO.

6. References

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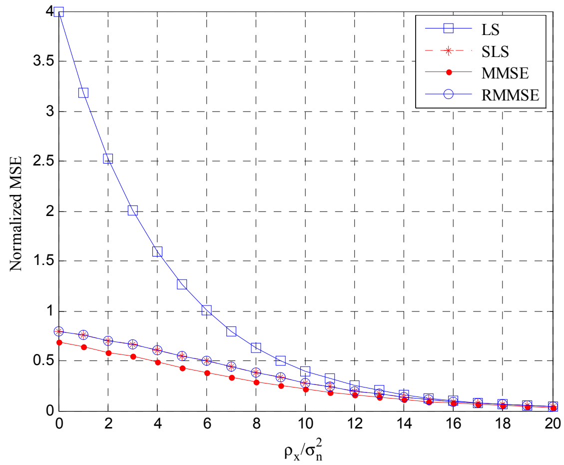

Fig. 1 Channel estimation MSE versus SNR for LS, SLS, MMSE and RMMSE estimators using orthogonal training sequences.

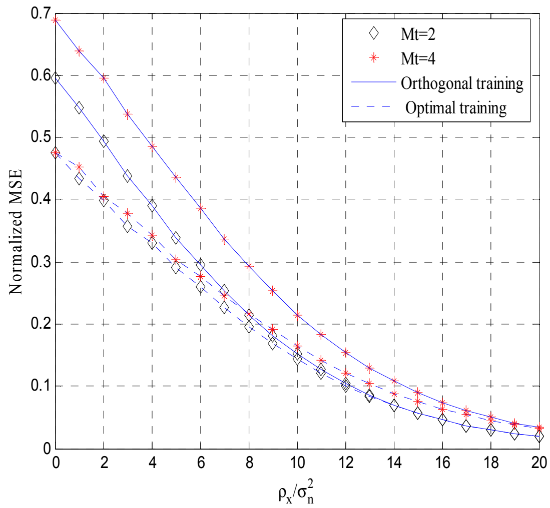

Fig. 2 Channel estimation MSE versus SNR for MMSE using orthogonal and optimal training sequences for different number of elements at MIMO transmitter arrays

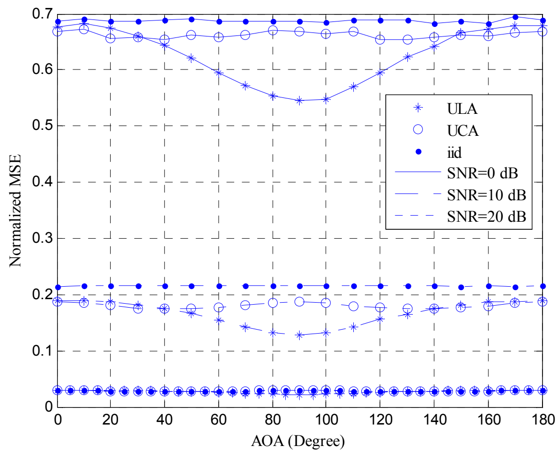

Fig. 3 Channel estimation MSE versus AOA for ULA-ULA MIMO and ULA-UCA MIMO systems employed at the receiver end with various SNR values.

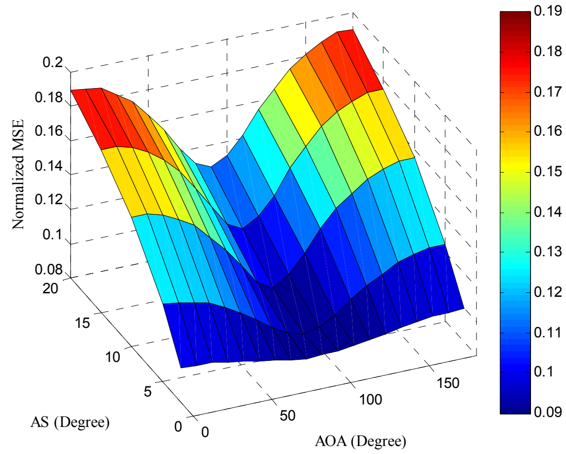

Fig. 4 Normalized MSE of MMSE channel estimator versus AOA and AS at SNR =10dB, in case of ULA- MIMO receiver.

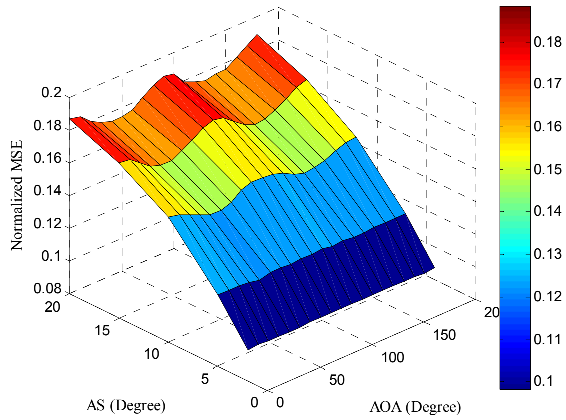

Fig. 5 Normalized MSE of MMSE channel estimator versus AOA and AS at SNR =10dB, in case of ULA-UCA MIMO system.

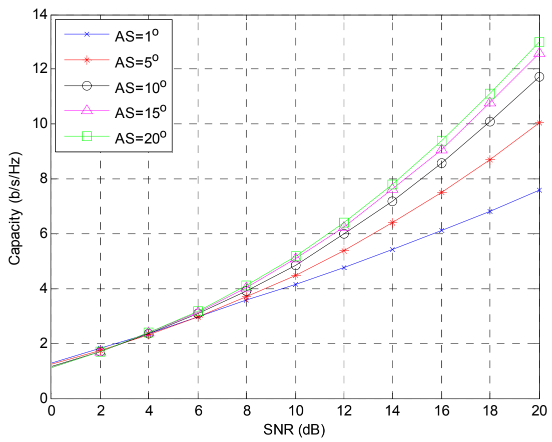

Fig. 6 Ergodic Capacity for UCA- MIMO systems with MMSE channel estimator versus SNR at different AS.