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An Improved Algorithm and It's Application to Sinusoid Wave Frequency Estimation

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Abstract. An improved algorithm based on Kay's estimator and its application to Sinusoid Wave frequency estimation are investigated. Firstly a new method of spectrum analysis is introduced, which has excellent performance in suppressing spectral leakage and the property of 'phase invariant' , then a hybrid All Phase Kay(ApIkay) algorithm is proposed ,which merges Kay 's estimator and phase unwrapping. The improved algorithm is applied to the frequency estimation of a sinusoid, the frequency performance is better than Kay and Ikay. When SNR(<7 dB), The simulation results show that the mean square error of the new frequency estimator is improved 4dB than Kay, 1 dB than Ikay. When SNR(>7 dB), the performance of ApIkay obtains CRLB quickly, and the performance is stable in the whole frequency range.

Keywords: frequency estimation, Kay, phase unwrapping, sinusoid, CRLB

1. Introduction

Estimating the frequency of a single sinusoid corrupted by additive, white, Gaussian noise (AWGN) is an important and classical problem in communications, radar and sonar signal Processing. Maximum likelihood (ML) frequency estimators in frequency-domain were studied by Rife and Boorstyn in [1], which has large complexity. The time-domain estimators in [2-8] are derived from the ML principle. Tretter proposed unwrapping the signal phase and performing linear regression to obtain a frequency estimate [2], but can only work well at high signal-to-noise ratio (SNR). Kay addressed the phase unwrapping problem by only considering the phase differences and presented a simple frequency estimation algorithm, namely, Kay's estimator [3], which can approach the Cramer-Rao lower bound (CRLB) at high SNR, but this estimation method has obvious threshold in reality application and has relation with the frequency. The performance becomes bad when frequency is close to 0, half of sample frequency, sample frequency. Ikay in [9] improved Kay's estimation performance. In this paper, we improve the Ikay further, ApIkay is proposed based on Kay's estimator and a new method of spectrum analysis and phase unwrapping, which has better performance that the MSE of frequency estimation improves about 4dB than Kay's and 1 dB than Ikay in the low SNR(<7 dB), and close to CRLB when (>7 dB) quickly.

2. A New FFT Spectrum Analysis

A novel algorithm of spectrum estimation is put forward in the literature [10], named ApFFT, which improves the data truncating way of traditional DFT spectrum analysis and reduces the leakage greatly. The block diagram is shown in the bottom of the figure 1.The 2N-1 order window function is the convolution of two same symmetric N order window.

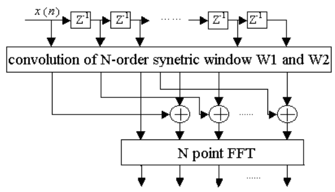

Figure 1. Figure 1 the diagram of ApFFT spectrum analysis

First, we deduce the amplitude of a signal consisting of a single frequency f0 .If the signal with single frequency is $x = e^{j\frac{2\pi f_0}{f_s}n}$, where f_0 is the signal frequency, f_s is the sample frequency.

To one sample point $x(N)$ in the time sequence, there are N vectors of N dimension including this sample point:

$$X_0 = [x(N) \quad x(N+1) \cdots x(2N-1)]^T$$
$$X_1 = [x(N-1) \quad x(N) \cdots x(2N-2)]^T$$
$$\cdots \cdots \cdots \cdots \cdots \cdots \cdots$$
$$X_{N-1} = [x(1) \quad x(2) \cdots x(N)]^T$$

Cycle shift every vector, shift the sample $x(N)$ to the first position of the sequence and get the other N vectors of N dimension:

$$X_0^{'} = [x(N) \quad x(N+1) \cdots x(2N-1)]^T$$
$$X_1^{'} = [x(N) \quad x(N+1) \cdots x(N-1)]^T$$
$$\cdots \cdots \cdots \cdots \cdots \cdots \cdots$$
$$X_{N-1}^{'} = [x(N) \quad x(1) \cdots x(N-1)]^T$$

We can get all phase data vector by adding N vectors aiming at $x(N)$

$$X_{AP} = \frac{1}{N}[Nx(N) \ (N-1)x(N+1)+ x(1)\cdots x(2N-1)+(N-1)x(N-1)]^T$$

According to the shift property of discrete Fourier transform, there has clear relationship between the $X_i^{'}$ （k） and X_i （k） , where $X_i^{'}$ （k） is the discrete Fourier transform of $X_i^{'}$(i=0,1,…N-1) and X_i （k） is the discrete Fourier transform of X_i(i=0,1,…N-1).

$$X_i^{'}(k) = X_i(k)e^{j\frac{2\pi ki}{N}} \tag{1}$$

ApFFT is made up of the sum of $X_i^{'}$ （k） ,so:

$$X_{AP}(k) = \frac{1}{N}\sum_{i=0}^{N-1} X_i^{'}(k) = \frac{1}{N}\sum_{i=0}^{N-1} X_i(k)e^{j\frac{2\pi ki}{N}}$$
$$= \frac{1}{N}\sum_{i=0}^{N-1}\sum_{n=0}^{N-1} e^{j2\pi\frac{f_0}{f_s}(N-i+n)} e^{-j\frac{2\pi}{N}kn} e^{j\frac{2\pi ki}{N}} \qquad = \frac{1}{N}e^{j2\pi\frac{f_0}{f_s}N}\sum_{i=0}^{N-1} e^{-j2\pi(\frac{f_0}{f_s}-\frac{k}{N})i}\sum_{n=0}^{N-1} e^{j2\pi(\frac{f_0}{f_s}-\frac{k}{N})n}$$
$$= \frac{1}{N}e^{j2\pi\frac{f_0}{f_s}N}\frac{Sin^2\pi N(\frac{f_0}{f_s}-\frac{k}{N})}{Sin^2\pi(\frac{f_0}{f_s}-\frac{k}{N})} \tag{2}$$

According to (2), the amplitude of all phase spectrums is as follows $\frac{1}{N}\left|\frac{\sin \pi N(\frac{f_0}{f_s}-\frac{k}{N})}{\sin \pi(\frac{f_0}{f_s}-\frac{k}{N})}\right|^2$ ， it is the square of traditional DFT frequency spectrum amplitude, which is benefit to reducing the spectrum leakage.

Another important character of ApFFT spectrum analysis is that its phase is constant and isn’t influenced by the frequency shift, so the phase needn’t to be corrected. That means the real phase of signal can be obtained by ApFFT spectrum analysis when the signal is non-integer truncated. The measured phase value and the real phase value have less error.

Take the signal cos(1.2×2π/6t+100π/180) as an example to search the reason of so little phase error about ApFFT spectrum analysis. We can get 11 samples:

-0.1736 -0.9903 -0.4384 0.7193 0.8829 -0.1736 -0.9903 -0.4384 0.7193 0.8829 -0.1736

The input signal of all phase is made up of 6 groups of N=6 samples. The first group consists of the last 6 samples among all 11 samples, the second group consists of another 6 samples which left shift 1 value, but -0.1736 should right shift to the first position, other groups could be get as the same way.

-0.1736	-0.9903	-0.4384	0.7193	0.8829	-0.1736
-0.1736	-0.9903	-0.4384	0.7193	0.8829	0.8829
-0.1736	-0.9903	-0.4384	0.7193	0.7193	0.8829
-0.1736	-0.9903	-0.4384	-0.4384	0.7193	0.8829
-0.1736	-0.9903	-0.9903	-0.4384	0.7193	0.8829
-0.1736	-0.1736	-0.9903	-0.4384	0.7193	0.8829

The phases of the samples of the 6 group of N=6 signals are :

180.0000	132.5107	-25.7699	0	25.7699	-132.5107
0	112.9053	51.7298	180.0000	308.2702	247.0947
0	112.4978	53.4788	180.0000	306.5212	247.5022
180.0000	88.4598	138.4375	0	-138.4375	-88.4598
180.0000	83.6275	161.4769	0	-161.4769	-83.6275
180.0000	72.5107	214.2301	180.0000	145.7699	287.4893

Because the frequency is 1.2, we should observe the second phase in every group. Three are bigger and three are smaller than the real phase (100) during 6 phases. Input data of ApFFT is the average of the above 6 groups' signals, phases are counteract each other, which make phase difference zero. So the phase got by ApFFT is the signal real phase. The result of experiment shows that when the signal is inter-period sampled, the phase got by ApFFT is perfect. In this case, the phase of signal got by ApFFT with window (kaiser(N,9.5) convolute Kaiser(N,9.5)) is as follows:

180.0000	100.0069	100.0004	180.0000	259.9996	259.9931

In this case, the real phase is 100, the measured phase is 100.0069, so the error is only 0.69%, and we can think them very similar. According to the above example, we can get the conclusion that all phase has perfect phase analysis property, especially when the signal is non-inter-period sampled, the phase analyzed by this method is almost the real value; while the phase analyzed by traditional method is deflect from the real value.

So a method of ApFFT phase difference is proposed in [10], which can estimate signal's parameters with less error, but has not good result in the low SNR.

3. Improved Algorithm based ApFFT and Kay

A mono-component sinusoid contaminated by AWGN can be modeled as:

$$r(n) = s(n) + w(n) = A\exp\{j\phi(n)\} + w(n), n = 1,...N$$

$$\phi(n) = 2\pi f_c nT + \theta_c$$

Where A is the amplitude, f_c and θ_c are frequency and initial phase, T is sample cycle.

We can get phase from (3) from the received signal:

$$\phi(n) = \arctan\left[\frac{\mathrm{Im}[r(n)]}{\mathrm{Re}[r(n)]}\right] \tag{5}$$

But this is not the true phase, the true phase $\phi_T(n)$ is obtained through phase unwrapping on the measured phase $\phi(n)$.

In the condition of noise, the relation between true phase and measured phase is as (6),where $k_n (k_n \in \mathbb{Z})$ is the number of period of the nth sample.

$$\phi(n) = \phi_T(n) - k_n 2\pi \tag{6}$$

The phase difference of adjacent samples $\Delta\phi$ is:6

$$\Delta\phi(n) = \Delta\phi_T(n) - (k_n - k_{n-1})2\pi \tag{7}$$

Where $\Delta\phi_T(n)=\phi_T(n)-\phi_T(n-1)$, $\Delta\phi(1)=\phi(1)$, $\Delta\phi_T(1)=\phi_T(1)$, $k_0=0$, $k_1=1$ .When $\phi_T(n)$ and $\phi_T(n-1)$ are in the same period $\Delta\phi(n)>0$,otherwise, $\Delta\phi_n<0$. Thus the true phase can be recovered according to the phase difference between adjacent samples. The true phase of the nth sample is:

$$\phi_T(n)=\phi(n)+k_n 2\pi,\begin{cases} if\ \Delta\phi(n)\geq 0, k_n=k_{n-1} \\ if\ \Delta\phi(n)<0, k_n=k_{n-1}+1 \end{cases} \tag{8}$$

The better the performance of phase unwrapping; the closer to π the phase difference of adjacent samples. By using this performance, an improved algorithm based on ApFFT and Kay is proposed, the step of this algorithm is as follow:

a) Obtain $R(k)$ by Performing Fourier transform on $r(n)$, then estimate the center frequency $\tilde{f}_c$ of $r(n)$,compute the shift value $f_0=f_s-\tilde{f}_c$,where f_s is the sampling frequency.

(i) Take $2N-1$ samples and shift to center frequency, another signal can be obtained:

$z(n)=r(n).*\exp(j2\pi \tilde{f}_c nT)$,where $2N-1$ is the samples length of $r(n)$.

(ii) Perform ApFFT on $z(n)$ in order to get phases $\phi_Z(n)$,the length of which is N.

(iii) Unwrap $\phi_Z(n)$ by using formula (8) to get the true phase $\phi_{ZI}(n)$.

(iv) Get Phase difference $\Delta\phi(n)$ through two slices of phase data with length N-1, which is shift only one point.

(v) Get the unbiased estimator of frequency $\hat{f}=\sum_{n=0}^{N-1}h(n)\Delta\phi(n)$,where $h(n)=\frac{\frac{3}{2}N}{N^2-1}\left\{1-[\frac{n-(\frac{N}{2}-1)}{\frac{N}{2}}]^2\right\}$.

(vi) Get the true frequency of $r(n)$: $f=\hat{f}-f_0$.

4. Prepare Your Paper Before Styling

Do some simulations By Matlab to test the improved algorithm. Take two groups of sinusoid wave as example, frequencies are 20Mhz and 40Mhz separately, sampling frequency is fs=100Mhz and the length of samples is 2*N-1=63, estimate frequencies of this two sinusoid waves by Kay' estimator and Ikay and ApIKay(in order to simple, we named the method of this paper ApIKay) , under the low SNR condition(SNR<7dB),do Monte-Carlo simulations to get mean square error. The number of simulation runs is set to 1000 for each case; the result is shown in table 1 and 2.

TABLE I. FREQUENCY ESTIMATION COMPARISON ($f_c=20MHz$, UNIT:KHZ)

	0dB	1 dB	2 dB	3 dB	4 dB	5 dB	6 dB
Kay	4331.4	3925.4	3281.8	2611.8	1957.5	1269.3	814.36
IKay	2249.1	1795.4	1433.5	1038.2	685.38	397.41	371.65
ApIkay	1759.1	1456.9	998.67	773.66	488.85	324.6	291.44
CRLB	215.4	191.9	171.1	152.5	135.9	121.1	107.9

TABLE II. FREQUENCY ESTIMATION COMPARISON ($f_c=40MHz$, UNIT:KHZ)

	0dB	1 dB	2 dB	3 dB	4 dB	5 dB	6 dB
Kay	6478.6	6353.4	6189.3	6171.0	5997.1	6080.0	5482.3
IKay	2291.4	1852.5	1395.2	1043.6	637.6	436.59	313.06
ApIkay	1757.8	1380.9	1040.4	789.5	440.91	323.70	269.92
CRLB	215.4	191.9	171.1	152.5	135.9	121.1	107.9

From Table1 and Table2, we can conclude that the mean square error（MSE） of IKay and ApIkay improve 3dB and 4 dB or so than that of Kay separately. From Table2, we can see that the MSE of Kay is very large when the signal frequency is close to 1/2 sampling frequency, while Ikay and ApIkay are far better than Kay. This is because Kay's estimator is sensitive to frequency. Now we compare the performance relationship between MSE and frequency by using three methods. Simulation conditions: SNR=2 dB, frequency step is 5 Mhz, the range of frequency is from 0 Mhz to 100 Mhz, there are 21 frequency points. Compute MSE of this frequency points, the result is shown in the figure 2. From Figure (2), we can see that the performance of Kay is bad when frequency is close to 50 MHz, while Ikay and ApIkay are better in the whole range of frequency. The mean MSE of Kay and Ikay and ApIkay during the whole frequency are 20.0001MHz and 1358.2 KHz and 1076.5 KHz, ApIkay's is most close to CRLB which is 171.07 KHz.

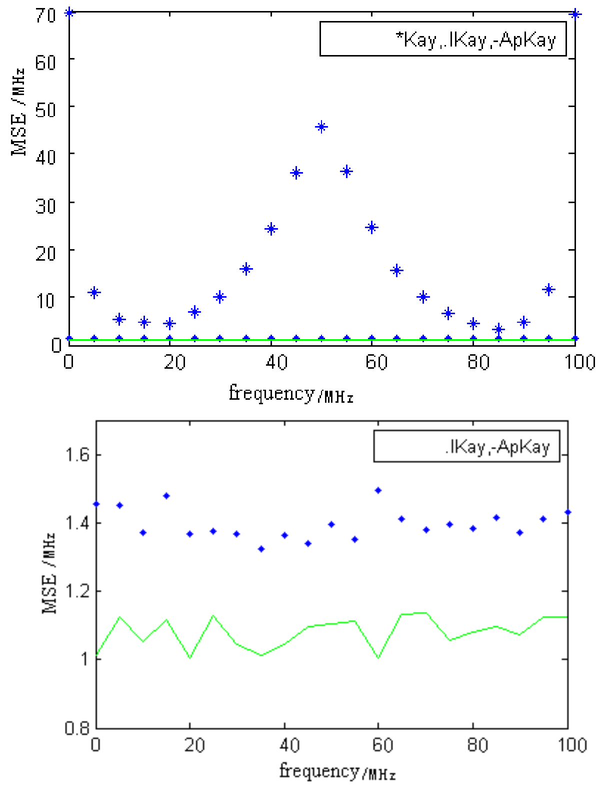

Figure 2. Performance relationship between MSE and frequency by using three methods (the following figure is zoom out figure)

When SNR>6dB, the estimate accuracy of ApIkay closes to CRLB. Take the above signal 20Mhz sinusoid wave as example, compute the MSE of the signal by using these three algorithms at different SNR. The result of simulation is as the Figure (3).

From Figure(3), We can draw a conclusion that ApIkay reduces the SNR threshold of Kay and Ikay, that means the MSE of ApIkay obtains CRLB at SNR=6dB,while Kay obtains CRLB at SNR = 10dB, Ikay obtains CRLB at SNR=8dB.

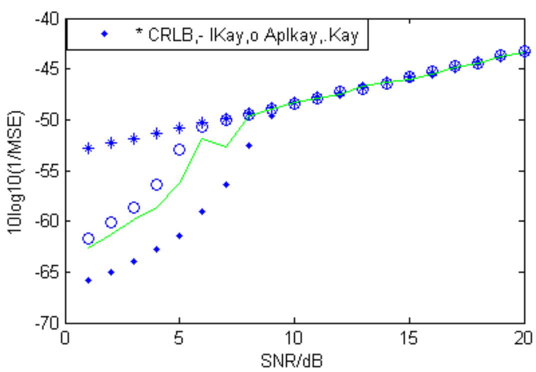

Figure 3. Performance of SNR threshold

Take another signal as example to test the frequency accurate by Kay,Ikay,ApIkay and ApFFT phase difference [10](named ApPD). The signal is complex exponent signal, fs is 32hz, we consider three cases about signal frequency, which is 6.7316hz, 12.2631hz, 15.7386hz separately. The SNR is 2dB. Signal length is N=256. The simulation result is as Table3.

TABLE III. FREQUENCY ESTIMATION COMPARISON

	2dB		
True frequency	6.7316	12.2631	15.7386
Kay	5.6289	5.8478	0.50065
IKay	6.7344	12.2642	15.7345
ApIkay	6.7341	12.2630	15.7371
ApPD	6.8527	11.9623	15.8916

5. Conclusion

This paper brings forward ApFFT, which has less leakage and high precision and phase invariant compared to the traditional spectrum analysis. An improved algorithm of sinusoid wave under the noise background is proposed, which has better frequency estimation characteristic, improves the SNR threshold of Kay, and the performance is stable in the whole frequency range, which is a problem in Kay. So the research in this paper can be used in parameter estimation in the area of communication and Radar.

6. Acknowledgment

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