

A Real-time Cycle-slip Detection and Repair Method for Single Frequency GPS Receiver

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Abstract—In GPS receivers, carrier phase measurement can be used to improve the receiver's position accuracy. In order to maintain the accuracy, cycle-slip must be detected and repaired instantaneously and accurately. This paper first analyzes the implementation of Doppler-aided cycle-slip detection and repair method. Then introduce a simplified oscillator model. Based on the oscillator model, a modified method is proposed, which avoids the influence of the local oscillator bias. Data tests show that the root mean square error of the time-difference measurement residual based on the proposed method is small enough for detecting and repairing the cycle-slip instantaneously.

Keywords: Doppler-aided, cycle-slip, single-frequency, GPS, real-time.

1. Introduction

With the continuous progress of GPS Modernization and GPS applications continue to expand, the economy, high precision and high reliability of the GPS receiver has become more and more popular. In order to obtain accurate positioning, carrier phase measurement is usually used in positioning. In addition to the initial integer ambiguity of the carrier phase measurement, cycle-slip is still a big challenge compared with the pseudo-range measurements. Cycle-slip is discontinuity of an integer number of cycles in the measured carrier phase resulting from a temporary loss-of-lock in the carrier tracking loop of a GPS receiver. The causes of the cycle-slip is listed as below[1]:

- 1) Cycle-slip is caused by obstructions of the satellite signal due to trees, buildings, bridges, mountains, etc.
- 2) Cycle-slip is a low signal-to-noise ratio (SNR) or alternatively carrier-to-noise-power-density ratio (C/N0) due to bad ionospheric conditions, multipath, high receiver dynamics, or low satellite elevation angle.
- 3) Cycle-slip is a failure in the receiver software which leads to incorrect signal processing.

The occurrence of cycle-slip affects not only the current measurement, but also the following epochs. It seriously degrades the positioning accuracy. In order to attain constant high-precision positioning result, cycle-slip must be detected and repaired or handled with carrier phase measurements at the data processing stage.

Currently, many methods are used to detect and repair cycle-slip, such as polynomial fitting, high-order difference method, combination method of pseudo-range and carrier phase, ionosphere residual method and so on[2]. But these methods have their own disadvantages: Polynomial fitting can be used for single or dual frequency measurements in post-processing, but it can't be used in real-time cycle slip detection. High-order difference method can't detect small cycle-slip. The ionosphere residual method must be used in dual-frequency receivers and cannot indicate on which channel the cycle-slip takes place. The combination method of pseudorange and carrier phase depends on the precision of pseudorange completely.

Doppler measurement is the instantaneous change rate of carrier phase. It is a very robust measurement. Therefore, Doppler measurement is an alternative way to detect and repair cycle-slip. However, in practice,

the oscillator is a non-ideal clock source. The deviation in oscillator may result in Doppler measurement error. The instantaneous clock deviation estimation is not easy. The oscillator's error of the receiver will be appeared in the Doppler-aided cycle-slip detection and repair method (DCDRM). Based on the relationship between Doppler measurements and carrier phase measurements, this paper proposes a new method called modified Doppler-aided cycle-slip detection and repair method (Modified DCDRM), which avoids using the corrected Doppler measurements and integration time to detect and repair the cycle-slip.

2. Real-Time Cycle-Slip Detection Technology

2.1 Doppler-aided cycle-slip detection and repair method

The carrier phase measurement equation can be written as[3]:

$$\begin{aligned}\Phi &= \Phi_u - \Phi_s + N \\ &= \frac{(r - I + T)}{\lambda} + \frac{c}{\lambda}(\delta t_u - \delta t_s) + N + \varepsilon_\phi\end{aligned}\quad (1)$$

where Φ is the measured carrier phase; Φ_u is carrier phase generated by receiver; Φ_s is carrier phase arriving from satellite; λ is the carrier wavelength; r is the geometry range from receiver to GPS satellite; I and T are the delay of L1 carrier phase due to ionosphere and troposphere respectively; c is the speed of light; δt_u is the bias in receiver clock; δt_s is bias of the GPS satellite clock; N is the initial integer ambiguity, ε_ϕ is phase noise. Make difference between adjacent epochs, the time-difference measurement of carrier phase is described as:

$$d\Phi = \frac{(dr - dI + dT)}{\lambda} + \frac{c \cdot (d\delta t_u - d\delta t_s)}{\lambda} + dN + d\varepsilon_\phi \quad (2)$$

where $d\Phi$ is the time-difference measurement between adjacent epochs; dI and dT are the variation of ionosphere and troposphere delay respectively; $d\delta t_u$, $d\delta t_s$ is the variation of local, satellite clock bias; dr is the variation of geometry range from receiver to GPS satellite; dN is the cycle-slip.

Doppler measurement is immune from cycle-slip. So dr can be derived from Doppler measurements at adjacent epoch.

$$dr = \lambda \hat{\Phi}_d(k) = -\lambda \int_{t_{d0}} f_{d0} dt \approx -\frac{f_{d0}(k-1) + f_{d0}(k)}{2} \lambda T_{sa} \quad (3)$$

where $\hat{\Phi}_d(k)$ is the variation of geometry range from receiver to GPS satellite in the form of phase; f_{d0} is the true Doppler frequency; T_{sa} is the true integration time in GPS time.

As revealed in (2), the dr should be removed to estimate the size of the cycle-slip. Using the (3), the time-difference measurement residual (TDMR) can be represented as:

$$\delta\Phi = d\Phi - \hat{\Phi}_d = dN + \varepsilon'_\phi \quad (4)$$

$$\varepsilon'_\phi = \frac{dr}{\lambda} - \hat{\Phi}_d - \frac{(dI + dT)}{\lambda} + \frac{c}{\lambda}(d\delta t_u - d\delta t_s) + d\varepsilon_\phi \quad (5)$$

where $\delta\phi$ is the TDMR;

The variation of atmospheric delay, satellite orbit bias, multipath, and receiver system noise are to be more or less below a few centimeters as long as the observation sampling interval is relatively short, which is much less than one cycle-slip. Once the TDMR of current epoch is much smaller or larger than the average of TDMR, we can say that there is a cycle-slip at current epoch.

Consider the first two moments of TDMR in (4):

$$E(\delta\Phi_k) = dN_k + E(\varepsilon'_{\phi_k}), \quad k=1,2 \quad (6)$$

$$\text{Cov}(\delta\Phi_k) = \text{Cov}(\varepsilon'_{\phi_k}) \quad (7)$$

where $E(\)$ and $\text{Cov}(\)$ are mathematical expectation and variance-covariance operators, respectively. Since there is no redundancy to carry out statistical testing in real-time operation for (6) and (7). They have to be calculated through adaptive estimation. The mean value of TDMR $\delta\phi$ and its root mean square error (RMSE) σ_k is

$$\bar{\delta\Phi}_k = \bar{\delta\Phi}_{k-1} + \frac{1}{k}(\delta\Phi_k - \bar{\delta\Phi}_{k-1}) \quad (8)$$

$$\sigma_k^2 = \frac{k-2}{k-1}\sigma_{k-1}^2 + \frac{1}{k}(\delta\Phi_k - \bar{\delta\Phi}_{k-1})^2 \quad (9)$$

where $\bar{\delta\Phi}_k$ is the mean value of $\delta\Phi$ from epoch 1 to k ; σ_k is the covariance of TDMR at epoch k . The detection of cycle-slip is based on (10)

$$|\delta\Phi_k - \bar{\delta\Phi}_k| \leq p \cdot \sigma_k \quad (10)$$

where p is a scale factor of the threshold value which can define the ability to detect the cycle-slip.

When cycle-slip is detected, the next step is to determine its size. Cycle-slip can be repaired by the simplest way when the sampling interval is short enough. That is

$$dN = \text{round}(\delta\Phi_k - \bar{\delta\Phi}_k) \quad (11)$$

where $\text{round}(\)$ is a mathematical function which gets the nearest integer of the variable. When p is larger than 6, Equation (11) can be used to detect and repair one cycle-slip.

2.2 Oscillator model

As the local clock source is non-ideal, it will introduce the oscillator error into TDMR, making it much tougher to detect and repair small cycle-slip. An oscillator model has to be introduced into the modified DCDRM to avoiding the oscillator error.

The performance of frequency source is described by its accuracy and stability. Ideal oscillator stays at its nominal frequency in the life cycle. In fact, due to resonator aging, environmental influences such as vibration, temperature, pressure and humidity, will bring systematic bias and random error to frequency source. It can modeled as [4]

$$f(t) = f_0 + \Delta f + (t - t_0)\dot{f} + \tilde{f}(t) \quad (12)$$

where f_0 is the nominal frequency; Δf is the frequency bias; \dot{f} is a frequency drift; \tilde{f} is a random frequency. Reference [5-7] point out that ordinary oscillator has good stability in a short time. And the main error of the source is frequency bias. The oscillator model can be simplified as

$$f_a = f_n + \Delta f = (1 + \beta)f_n \quad (13)$$

where f_n is the nominal frequency; f_a is the actual frequency; β is a scale factor of the frequency bias. The relationship of sampling interval T_{sn} which is timing at the nominal frequency and the actual time T_{sa} is listed:

$$T_{sa} = T_{sn} / (1 + \beta) \quad (14)$$

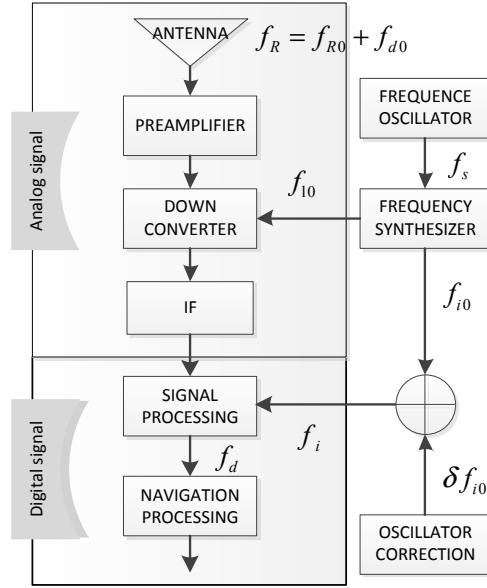


Figure 1. Generic receiver functional block diagram

2.3 Modified DCDRM

System level functional block diagram of a generic receiver is shown in Fig.1. The generic receiver consists of the following 8 function blocks[8]: antenna, preamplifier, reference oscillator, frequency synthesizer, down-converter, an intermediate frequency (IF) section, signal processing and navigation processing. The radio frequency signal down-converts to intermediate frequency signal. And then in the digital signal part, the intermediate frequency converts to base band signal.

Consider an ideal oscillator model, which means $f_s = f_n$, then the oscillator correction will be zero. Then the Doppler measurement will be

$$f_d = f_R - f_{i0} - f_{i0} = f_R - f_{R0} = f_{d0} \quad (15)$$

where f_d is the Doppler measurement based on f_n ; f_R is the received satellite signal frequency, which contains the satellite sending frequency f_{R0} and the true Doppler frequency f_{d0} , and $f_R = f_{R0} + f_{d0}$; f_{i0} and f_{i0} are the local oscillator frequency used to down-convert the satellite signal, and $f_{i0} + f_{i0} = f_{R0}$.

When the frequency source is not an ideal frequency source, according to (13), there will be a small deviation Δf , and $f_s = (1 + \beta) f_n$. All the frequency based on f_n will be biased. The relationship between the satellite signal and the local signal will be:

$$f_R = f_{R0} + f_{d0} = (1 + \beta)(f_{i0} + f_{i0} + \delta f_{i0} + f_d) \quad (16)$$

So the true Doppler frequency will be

$$f_{d0} = (1 + \beta)(f_{R0} + \delta f_{i0} + f_d) - f_{R0} \quad (17)$$

According to (1), TDMR can be expressed as

$$\begin{aligned} d\Phi(k) &= \Phi(k) - \Phi(k-1) \\ &= (\Phi_u(k) - \Phi_u(k-1)) - (\Phi_s(k) - \Phi_s(k-1)) \\ &= T_{sa} f_{R0} - (\Phi_s(k) - \Phi_s(k-1)) \end{aligned} \quad (18)$$

where $\Phi_u(k)$, $\Phi_s(k)$ are carrier phase generated in receiver and carrier phase arriving from satellite at epoch k ; T_{sa} is the actual sampling interval. From (3) and (17), $\hat{\Phi}_d(k)$ can be represented as

$$\begin{aligned}
\hat{\Phi}_d(k) &= -\int f_{d0} dt \approx -\frac{f_{d0}(k-1) + f_{d0}(k)}{2} T_{sa} \\
&= -[(1+\beta)(f_{R0} + \delta f_{i0} + \frac{1}{2}(f_d(k-1) + f_d(k))) - f_{R0}] \cdot T_{sa} \\
&= -(f_{R0} + \delta f_{i0} + \frac{1}{2}(f_d(k-1) + f_d(k))) T_{sa} + f_{R0} \cdot T_{sa}
\end{aligned} \tag{19}$$

According to (18) and (19), we have

$$\begin{aligned}
\delta\Phi &= d\Phi - \hat{\Phi}_d \\
&= (f_{R0} + \delta f_{i0} + \frac{1}{2}(f_d(k-1) + f_d(k))) T_{sa} - (\Phi_s(k) - \Phi_s(k-1))
\end{aligned} \tag{20}$$

From (20), we find that in the calculation of TDMR, it uses the raw data of carrier phase arriving from satellite, Doppler measurement based on f_n obtained from the signal processing block and the nominal sampling interval. It avoids correcting the local oscillator. And also it avoids introducing the oscillator error into the TDMR. It simplifies the computation and obtains a higher accuracy TDMR.

3. Test Result

In order to illustrate the performance of our approach, we have tested it with data sets in static and kinematic modes.

Static mode test is carried out in May 27, 2011 at Yuquan Campus of Zhejiang University, using a dual frequency receiver. We analyze the satellite PRN3's raw data. The mean value and the RMSE of TDMR are shown in Table I.

Kinematic mode test is based on a signal simulator. It simulates two type kinematic motions. Group one is uniform linear motion with the relative speed of -500 m/s. The other is linear motion with a constant acceleration 2m/s². And the result is shown in Table II.

TABLE I. THE TDMR IN STATIC MODE TEST

Sampling interval (s)	0.1	0.5	1	2	10
Mean (cycle)	0.0482	0.2406	0.4815	0.9355	5.2413
RMSE(cycle)	0.0169	0.0656	0.1652	0.5141	4.1215

TABLE II. THE TDMR IN KINEMATIC MODE TEST

Sampling interval (s)		0.1	0.5	1	2	10
Group one	Mean	0.0503	0.2495	0.5077	1.0240	5.0081
	RMSE	0.0261	0.1212	0.3238	0.9628	8.8139
Group two	Mean	0.0308	0.1456	0.2879	0.5168	5.3696
	RMSE	0.0300	0.1576	0.4671	1.1852	8.0267

Table I shows that the shorter the sampling interval is, the smaller the RMSE of the TDMR is. That is because time-difference measurement is not a linear function of Doppler frequency. It is an integration of the Doppler frequency. While in this method, an approximation is made by using the trapezoidal integration method, which makes the RMSE of TDMR increases larger as sampling interval increases.

Compared with Table I, Table II shows that the noise of TDMR in kinematic mode is significantly larger than in static mode at the same sampling interval.

The distribution of TDMR without cycle-slip is shown in Fig.2, there are 5000 samples at 0.1s sampling interval in each graph. Fig.2 shows that the TDMR of the three tests satisfy none-zero means Gaussian distribution. Over 98.2% of the noise of TDMR is in 3 times σ_k .

In the static mode test, one cycle-slip is manually inserted into the carrier phase measurements of satellite PRN3 at the time of 90 seconds. And the TDMR is shown in Fig. 3(a) and (b). It shows that TDMR of the epoch which has an occurrence of cycle-slip is much larger than the others. The same work is done on Kinematic mode tests at 1 second sampling interval. The results are shown in Fig.3(c) and (d). The size of the cycle-slip can be directly determined by rounding $\delta\Phi_k - \bar{\delta\Phi}_k$.

4. Conclusion

In this paper, we first analyze the implementation of DCDRM. Then a modified method is proposed based on a simplified oscillator model. In this method, the estimation of oscillator bias is removed. The test result shows that the mean value of TDMR is relatively robust, and its RMSE is rarely small at high sampling rate. The cycle-slip can be detected and repaired by rounding $\delta\Phi_k - \bar{\delta\Phi}_k$ as long as sampling interval is short enough. Test result also shows that the RMSE of TDMR is larger in kinematic than in static mode. We can choose high sampling rate when the receiver is in kinematic mode.

5. Acknowledgment

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6. References

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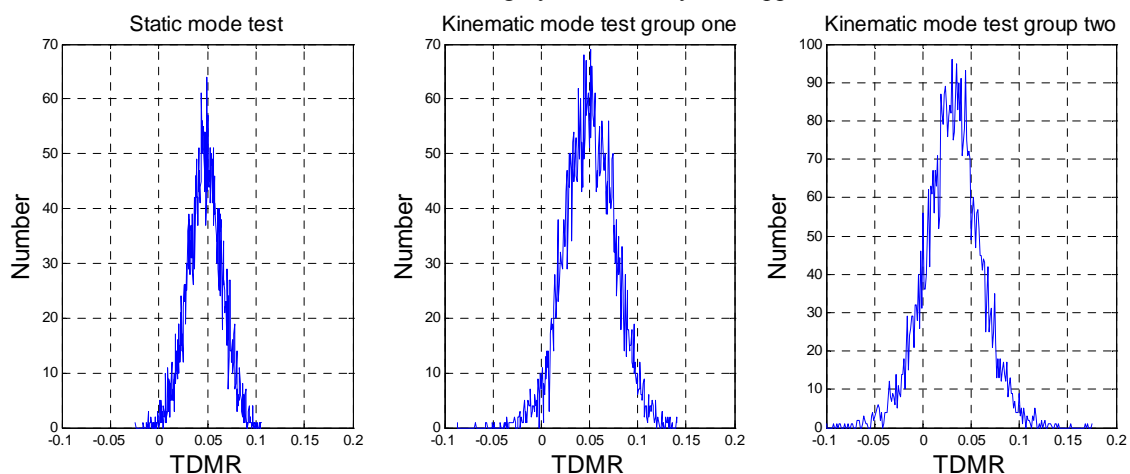


Figure 2. The distribution of TDMR in static mode、 uniform linear motion mode with the relative speed of -500 m/s and linear motion with a constant acceleration 2m/s². All the three charts show that the TDMR of the three tests satisfy none-zero means Gaussian distribution.

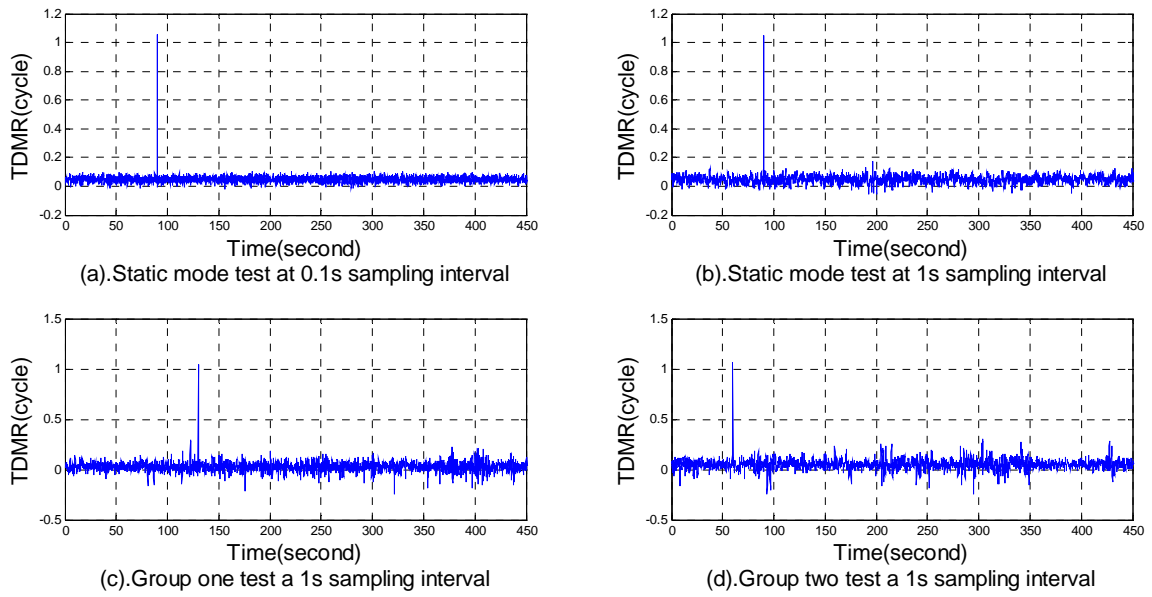


Figure 3. TDMR with one cycle-slip. (a) , (b) show one cycle-slip in TDMR at 0.1s and 1s sampling interval in static mode. It shows that the RMSE of TDMR increases as sampling interval increases. (c), (d) show one cycle-slip in TDMR at 1s sampling interval in Group one and Group two mode. It shows that RMSE of TDMR is much larger in kinematic mode than static mode. From all four charts, the one cycle-slip is much larger than the RMSE. We can directly determine the cycle-slip by rounding the TDMR.