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Evolutionary Public Goods Game in Group Structured Population

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Abstract—Evolutionary dynamics are strongly affected by population structure. The outcome of an evolutionary process in a well-mixed population can be very different from that in a structured population. This paper refers to an evolution model of group structured population. In this model, both individuals' strategies and population structure are subject to evolutionary updating. We specially focus on the evolution of cooperative behavior and study the dynamics of evolutionary public goods game in group structured population. Some key parameters affecting the track of the population's evolution and various dynamic natures shown during the process of evolution are studied by approaches combining theoretical analysis and computer simulations. We found that the co-evolution of individuals and groups structure is able to promote cooperation in public goods game. The model advised by us also can be extended and provide new research ideas for more other evolutionary multiplayer game.

Keywords-public goods game; evolutionary game; multiplayer game; structured population; groups structure; evolution of cooperation

1. Introduction

The geometry of human populations is determined by the associations that individuals have with various groups or sets. We always interact with other people by participating in activities or joining in institutions. Each person belongs to several groups. Such groups can be defined, for example, by working relationship, living area, social network or blood relationship and so on. There can be groups within groups. For example, the employees of the same company work on different subjects and take different departments. These group memberships determine the structure of human society. This structure not only specifies who meets whom but also defines the frequency and context of meetings between individuals.

People pay a keen attention to the activities of others and consider whether their success is correlated with belonging to particular groups. It is therefore natural to assume that people do not only imitate the behavior of successful individuals, but also try to adopt their group memberships. Therefore, the evolution of human population should include updating of strategic behavior and of group memberships. In the same way as successful strategies spawn imitators, successful groups attract more members. If we allow groups associations to change, then the structure of the population itself is no more static, but a consequence of evolutionary dynamics.

There have been many attempts to study the effect of population structure on evolutionary and ecological dynamics. These approaches include spatial models in ecology [1–8], viscous populations [9], spatial games [10–15], and games on graphs [16–19]. However, most of the studies have not related to the co-evolution of individuals and the structure of population.

The population model studied by us called group structured population in which individuals and population structure co-evolve under the influence of feedback from each other. To strengthen the significance of our study, we specially focused on the evolution of cooperative behavior and performed our study aiming at the multiplayer game which has general realistic significance but is less concerned because of the complex situations inside. We explored the process of the co-evolution. Some key parameters influencing the track of the evolution and various dynamic nature showed during the process of evolution have been studied. We got conclusion that the co-evolution of individuals and groups structure can promote cooperation in public goods game. In addition, though the starting point of our model is for human society, our framework is applicable to more other fields. For example, in animal population groups can denote living at certain locations or foraging at particular places. Any individual can belong to several groups. Offspring might inherit the set memberships of their parents.

2. Traditional Public Goods Game

In the traditional Public Good Game, there are N players, and each player has an amount c ('cost') fund available for investment to public funds. Players are told that the public funds will be multiplied by an enhancement factor r ($1<r<N-1$) and the result is equally distributed between all N members of all. Each one must decide whether he is going to contribute the fund of himself.

One who contributes is called Cooperator and who do not contribute is called Defector. Defectors get the same benefit of the cooperators at no cost. Thus, defectors as individuals who have a highly adaptability will be imitated by more and more individuals. As more and more individuals adopt defection, one conflict arises: along with the public funds becoming less and less, everyone's payoff will be less and less too. An extreme case is that none of the players contributes, and then everyone's payoff is zero. This makes the "Tragedy of the Commons" [20].

But the real experimental results are quite different from these theoretical analyses [21]. In order to solve the conflict between theory and experiments, a lot of incentive mechanisms for cooperation have been proposed. Hauert et al. reported volunteering can promote cooperation in public goods game [21]. Szabó et al. studied the volunteering mechanism in spatial [22]. Punishment is proved that it can promote cooperation [23-27]. Recently, it is found that social diversity promotes the emergence of cooperation in public goods games [28-32].

3. The Model of Evolutionary Public Goods Game in Group Structured Population

3.1 Introduction of the model

Based on the observation of the real world, we modeled a group structured population. An example is showed in Fig. 1 where circles represent individuals, squares represent groups, and links represent group associations. This figure shows a population of 10 individuals distributed over 5 groups. Each individual can take one of two strategies: "cooperation" represented by blue and "defection" represented by red. Individuals interact with others who belong to the same group. If individuals have several groups in common, they interact several times. Each group can accommodate any number of individuals and can be empty too.

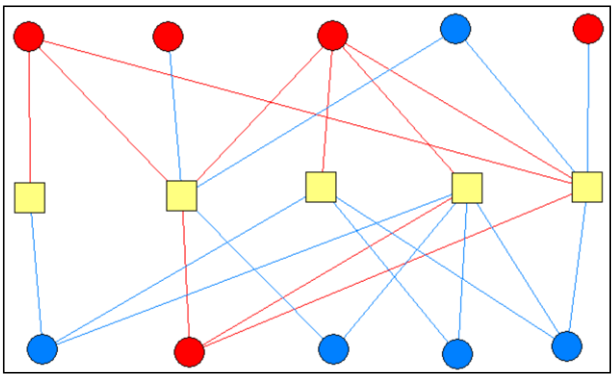

Figure 1. Example of population and its structure

Interactions lead to payoff from an evolutionary game. The payoff of the game is interpreted as fitness [33-37]. Individuals update stochastically in discrete time steps. Payoff determines fitness. Successful individuals are more likely to be imitated by others. An imitator not only takes the successful individual's strategy but also adopts the successful groups associations. Thus, both the strategy and the group memberships are subject to evolutionary updating.

Through the analysis in section 2, it is obviously that the result of evolutionary public goods game in a well-mixed population is that cooperators become extinct. Here raised a key question whether the evolution of group structure will be an incentive mechanism for cooperation behavior making cooperators not to perish or even beat defectors.

3.2 Theoretical analysis

Consider a population of N individuals distributed over M sets. In order to get simplicity of mathematical calculations, each individual is configured belonging to exactly K sets, where $K \le M$. Additionally, each individual has a strategy $s_i \in \{0,1\}$, referred to as cooperation, 1, or defection, 0. A state S of the system is given by a vector s and a matrix H. Vector s is the strategy vector; its entry s_i describes the strategy of individual i. Thus, $s_i = 0$ if i is a defector and it is 1 if i is a cooperator. H is a $N \times M$ matrix whose *ij*-th entry is 1 if individual i belongs to group j and is 0 otherwise. Therefore, row i of H gives the group memberships of individual i. Considering that individuals interact as many times as many groups they have in common, we can now write the fitness of individual i as (1) where $1 \le j \le M; 1 \le l \le N$.

$$Pay_i = \sum_j \left[H_{i,j} \left(\frac{rc\sum_l H_{l,j} s_l}{\sum_l H_{l,j}} - cs_i \right) \right] \tag{1}$$

Term $\sum_l H_{l,j} s_l \Big/ \sum_l H_{l,j}$ in (1) represents the cooperator proportion of group j. Set $R_j = \sum_l H_{l,j} s_l \Big/ \sum_l H_{l,j}$, we can get (2) from (1).

$$Pay_i = c\left(r\sum_j H_{i,j} R_j - s_i K \right) \tag{2}$$

The total payoff of cooperators and defectors are $Pay_C = \sum_i s_i \cdot Pay_i$ and $Pay_D = \sum_i (1 - s_i) \cdot Pay_i$ where $1 \le i \le N$. Because the total number of cooperators and defectors are $\sum_i s_i$ and $N - \sum_i s_i$, the average payoff of cooperators and defectors are respectively (3) and (4).

$$\overline{Pay_C} = \frac{\sum_i s_i \cdot Pay_i}{\sum_i s_i} \tag{3}$$

$$\overline{Pay_D} = \frac{\sum_i (1 - s_i) \cdot Pay_i}{N - \sum_i s_i} \tag{4}$$

Only $\overline{Pay_C} > \overline{Pay_D}$ is satisfied will cooperators be selected over defectors. Supposing (3) > (4), we get:

$$N\sum_i s_i \cdot Pay_i > \sum_i s_i \sum_i Pay_i \tag{5}$$

Combining (2) and (5), we get (6) where $1 \le d \le N$.

$$\frac{r}{K} > \frac{N\sum_i s_i - \sum_{d,i} s_i s_d}{N\sum_{i,j} s_i H_{i,j} R_j - \sum_{d,i,j} s_d H_{i,j} R_j} \tag{6}$$

Let the state of system be represented by (s, H). And $\sum_i s_i = x$. Therefore, we can get analysis for terms in (6) as followings.

Firstly for $\sum_{d\,i} s_i s_d$, its result of calculation is:

$$\sum_{d,i} s_i s_d = x^2 \tag{7}$$

For $\sum_{i,j} s_i H_{i,j} R_j$, it can be thought as that through all the groups for group j R_j is accumulated for n_j^c times. $n_j^c = \sum_i H_{i,j} s_i$ is the number of cooperators in group j, so we get:

$$\sum_{i,j} s_i H_{i,j} R_j = \sum_j n_j^c R_j \tag{8}$$

For $\sum_{d,i,j} s_d H_{i,j} R_j$, there is an equation $\sum_{d,i,j} s_d H_{i,j} R_j = \sum_d s_d \sum_{i,j} H_{i,j} R_j = x \sum_{i,j} H_{i,j} R_j$ in which $\sum_{i,j} H_{i,j} R_j$ can be thought as that through every group for group j R_j is accumulated for n_j times. $n_j = \sum_i H_{i,j}$ is the total number of members of group j, so we can get $\sum_{d,i,j} s_d H_{i,j} R_j = x \sum_j n_j R_j$. Because $n_j = \sum_i H_{i,j}$, $R_j = \sum_l H_{l,j} s_l \Big/ \sum_l H_{l,j}$ and $n_j^c = \sum_i H_{i,j} s_i$, then $\sum_{d,i,j} s_d H_{i,j} R_j = x \sum_j n_j^c$. Additionally, because each individual is set belonging to exactly K sets, $\sum_{d,i,j} s_d H_{i,j} R_j$ can be translated into:

$$\sum_{d,i,j} s_d H_{i,j} R_j = x \sum_j n_j^c = x \cdot Kx = Kx^2 \tag{9}$$

Combining (6), (7), (8) and (9) we can get:

$$\frac{r}{K} > \frac{Nx - x^2}{N \sum_j n_j^c R_j - Kx^2} \tag{10}$$

Setting $R = \frac{x}{N}, \delta_j = \frac{R_j}{R}$ we can translate (10) into (11).

$$\frac{r}{K} > \frac{x(1-R)}{\left(\sum_j n_j^c \delta_j - Kx \right) R} \tag{11}$$

δ_j can be thought as the deviation of R_j from R. When δ_j is closer to 1 the deviation is smaller. Given $\sum_j n_j^c = Kx$, we propose a parameter Δ satisfying $\sum_j n_j^c \delta_j = \Delta \cdot \sum_j n_j^c = \Delta \cdot Kx$. So we can do deduction as following:

$$\frac{r}{K} > \frac{x(1-R)}{(\Delta \cdot Kx - Kx) R}$$

$$r > \frac{(1-R)}{(\Delta - 1) R}$$

If $\Delta - 1 < 0$, $r > (1-R)/(\Delta - 1)R$ is always true. If $\Delta - 1 > 0$, then $\Delta > 1 + 1/r(1/R - 1)$. That is to say when $\Delta < 1$ or $\Delta > 1 + 1/r(1/R - 1)$ is true, selection will favor cooperators. Δ largely reflects the heterology of cooperators' distribution. As the heterology get larger Δ value will be larger. Essentially, Δ is a parameter which represents mathematical coupling relationship between global system and local system. Since lack of mathematical tools, this conclusion just proved the possibility of cooperators being favored. Nevertheless, compared to the traditional negative results this conclusion is sill optimistic. To further understand the nature of the model, our work done through computer simulations is reported as below.

3.3 Computer simulations

1) *Evolutionary algorithm of the simulation:*

In each discrete time steps, several groups are chosen randomly to perform games. Payoff determines fitness. After a round of games, individuals update synchronously. The updating includes two aspects: group associations and individual strategy. The updating of group associations is before the strategy updating.

While an individual is updating his group associations, he will join in new groups with a probability P_1 and break away from old groups with probability P_2. How to choose group associations to add or delete is based on the welfare of the groups is good or bad. Individuals tend to join in high-welfare groups and get out of low-welfare group. P_1 and P_2 can be regarded as the rate of group structure updating. The information about group welfare is localized. Individuals are only aware of groups owned by themselves or by their peers.

While an individual are updating his strategy, he will choose a group owned by him and having the highest welfare, then he randomly adopt the strategy of an individual in this group with probability P_3. P_3 can be regarded as the rate of individual strategy updating.

2) Results and discussions:

We verified the model's capability of prompting cooperation. The experimental environment is initialized with conditions $r=2$ and $N=100$ which includes $n_C=10$ cooperators and $n_D=90$ defectors. Initially individuals randomly select several group associations. In order to ensure a comprehensive study, we collected data from processes of evolution under setting M to equal to from 1 to 9. The experiment results are shown in Fig. 2.

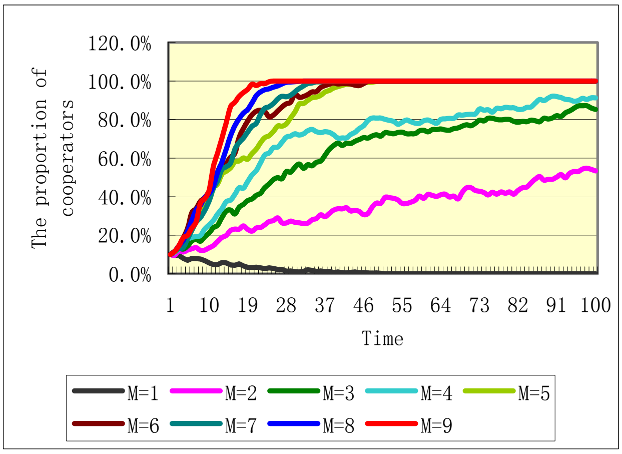

Figure 2. Time sequences of cooperators' proportion for different M values during the processes of evolutions. The curves shown in this figure are averaged through 1000 experiments.

Through comparing the curves in Fig. 2, we can see that both of the rates of the cooperators' expansion and the cooperators' final status are proportional to M value. For this phenomenon, we believe that is caused by two factors.

Factor 1 is that for a fixed population an appropriate number of groups will help the cooperators distribute heterogeneously. The heterology of cooperators' distribution is relevant to the Δ proposed in section 3.2. So, according to the conclusion in section 3.2, as the heterology get larger Δ value will be larger and then cooperators will be more favored. Factor 2 is the difference between game's parameter r and the average number of players involved in one game N/M. The smaller formula $N/M-r$ is the more favored cooperators will be. With N and r being static, $N/M-r$ is inversely proportional to M. We argue that when M value is at a lower lever of the middle range, factor 1 plays a major role and when M ranges at higher levels, factor 2 plays a major role.

When $M=1$ this model degenerates into a well-mixed population. The experimental data proved that under this condition cooperators get disappeared gradually. It matches the analysis in section 2.While $M \geq 5$ cooperators could beat defectors completely. When $N=100$ and $M=5$ the average number of players who join a game is 20. That is to say under the condition that the number of players is ten times r at least cooperators can overwhelmingly dominant. For the target of cooperation evolution we have achieved significant results.

4. Conclusion

In the evolutionary model introduced by our study, the evolution of population includes updating both of strategic behavior and group memberships. We aimed at the conflict between the theoretical analyses and the real experimental results in the research of public goods game, and studied the dynamics of evolutionary public goods game in group structured population. Our mathematical analysis and computer simulations demonstrate that: the evolution of group structured population which is based on the observation of the real world is an effective incentive mechanism for cooperation. The nature of this mechanism is like the Spatial Reciprocity theory or some other similar theories [10, 17, 38, 39]: the structure of population can make cooperators form "clusters". In our model, places which help cooperators gather just are the groups.

Although this paper only deals with the public goods game, the research ideas and methods also can be extended for other types of multiplayer game, e.g. proportional game and queuing game. However, because the context of multiplayer game is much more complex than 2-player game, it is difficult to present a unified paradigm for all of the multiplayer games. So the extensions of this model could not just depend on some simple substitutions but need to do more detailed works.

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