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Constitutive Equations for Metallic Crystals under Very High Strain Rates Loadings

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Abstract. Constitutive equations are being considered for metallic crystals under very high strain rates loadings. An analytical solution of the steady state propagation of plastic shocks is proposed. Constitutive relations are provided for the kinematics and kinetics of metals with micron-scale grains. The effect of strain, strain rate are being considered. But the effect of temperature is neglected.

Keywords: Constitutive equations, High strain rate, Shock loading, Viscoplastic.

1. Introduction

The responses of metals subjected to high stress states and very high rates of deformation (10^4 s^{-1}) are an area of active research. There are many engineering applications of metals related to high strain rate deformation such as high speed machinery, high speed transportation vehicles and spacecraft, high-speed impact phenomena, metal cutting and earthquake phenomena. In order to optimize the performance of metals undergoing plastic deformation at such dynamic loading conditions, it is necessary to understand its physical mechanism, which is different with the counterpart at low strain rates.

Moreover, when the strain rate exceeds 10^4 s^{-1}, many investigators found experimentally that the flow stress increases dramatically in many metals and alloys [1-5].

Viscous drag effects on dislocation movement are thought by some researchers [6] as the main origin of the increase of flow stress at such high strain rate regime at such high strain rate regime. However, Armstrong and co-workers [7, 8] proposed that such flow stress increase is caused by the dislocation generation at the shock front, not by a retarding effect of dislocation drag.

A shock, also called a shock wave, is a propagating surface at which the displacement is continuous but the mass density, particle velocity, stress, and other field variables are discontinuous. The work presented here is intended to address the viscoplastic deformation of metals in shock waves. However, the models may also be applied to ‘‘shockless” high-strain-rate phenomena (e.g., quasi-isentropic compression waves).

Constitutive models for the dynamic strength of metals have been constructed specifically for the shock loading regime. For example, Steinberg et al. [9] proposed a model wherein the yield strength is essentially framed as a first-order Taylor expansion in pressure about the ambient state. Clifton [10] investigated constitutive equations elastic visco plastic transient shock wave. Molinari and Ravichandran developed analytical method of Clifton for steady plastic shock waves [11]. Physically-based model is developed to address slip in polycrystalline metals and alloys subjected to shock loadings by [12].

The present work is an attempt to develop a unified physically based constitutive model for fcc metals under steady plastic shock waves. This formulation is derived by incorporating the viscous drag effects and by relating the entropy generation to the glide and accumulation of dislocations. Such relationship is based on the theory of irreversible thermodynamics which is widely employed in chemical and mechanical

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engineering and used recently by Huang and co-workers [13]. The fcc metal is 6061-T6 aluminum and the effect of temperature and grain size is neglected.

2. Theory

2.1. Kinematics

For uniaxial deformation and motion, the propagation of plane longitudinal waves, can be expressed in the especially simple lagrangian form:

$$x_1=X_1 + U(X,t), \quad x_2=X_2, \qquad x_3 = X_3 \tag{1}$$

Which X is a Cartesian reference frame, x is current configuration and U is displacement of X from its reference position in the positive x direction. But in steady shock wave with the additional constraint, the disturbance propagates unchanged in form and at a constant velocity. This means that the displacement

In eq. 1 can be rewritten as:

$$U(X,t) = U(Z) \ \text{ where } \ Z=X-Ct \tag{2}$$

For the special case of uniaxial strain in X1 direction, the deformation gradient has the form:

$$F = F^p F^e \tag{3}$$

With:

$$\boldsymbol{F}^e = \begin{bmatrix} \lambda_1^e & 0 & 0 \\ 0 & \lambda_2^e & 0 \\ 0 & 0 & \lambda_2^e \end{bmatrix}$$

$$\boldsymbol{F}^p = \begin{bmatrix} \lambda_1^p & 0 & 0 \\ 0 & \lambda_2^p & 0 \\ 0 & 0 & \lambda_2^p \end{bmatrix} \tag{4}$$

$$\boldsymbol{F} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where λ_1^e, λ_2^e are elastic stretches and λ_1^p, λ_2^p are plastic stretches. Using Plastic incompressibility:

$$\lambda_1^p (\lambda_2^p)^2 = 1 \tag{5}$$

For large deformation, the elastic strain can be neglected against plastic strain. Plastic strain can be written as:

$$\varepsilon_1^p = 1/2[(\lambda_1^p)^2 - 1] \tag{6}$$

$$\varepsilon_2^p = 1/2[(\lambda_2^p)^2 - 1] = 1/2[\tfrac{1}{\lambda_1^p} - 1] \tag{7}$$

2.2. Constitutive Equations

The Lagrangian form of the equation of conservation of linear momentum is:

$$\frac{\partial \sigma_1}{\partial X} = -\rho_0 c \frac{\partial v_1}{\partial t} \tag{8}$$

v_1 is the particle velocity, c is the propagation of longidutinal shock wave velocity, ρ_0 is the mass density in the reference configuration, and σ_1 is the component of the Piola–Kirchhoff stress tensor in the wave propagation direction. Using axisymmetry, the stress tensor can be expressed in the form:

$$\sigma = \sigma_1 e_1 \otimes e_1 + \sigma_2 e_2 \otimes e_2 + \sigma_2 e_3 \otimes e_3 \tag{9}$$

The kinematic compatibility equation has the form:

$$\frac{\partial v_1}{\partial X} = -c \frac{\partial \lambda_1^p}{\partial t} \tag{10}$$

The conservation energy law has the form of:

$$\frac{1}{2}\rho_0 v_1^2 + \sigma_1 \frac{\partial \varepsilon_1^p}{\partial t} + 2\sigma_2 \frac{\partial \varepsilon_2^p}{\partial t} = \tau \frac{\partial \gamma^p}{\partial t} \tag{11}$$

The viscoplastic flow rule has the following functional form (Prantel-Russ relation):

$$\gamma^p = \int \dot{\gamma^p} dt = -\frac{3}{2} \ln (\lambda_1^p) \tag{12}$$

τ is the maximum shear stress, which in the current configuration is given by:

$$\tau = \frac{\sigma_1 - \sigma_2}{2} \tag{13}$$

The relation between strain rate and mobile dislocation density is expressed by:

$$\dot{\gamma^p} = b\rho_m v + bl^* \frac{d\rho^+}{dt} \tag{14}$$

$bl^* \frac{d\rho}{dt}$ is for high strain rate loading, especially shock loading. b is burgers vector, ρ_m is mobile dislocation , v is velocity dislocation, l^* displacement of dislocations and $\frac{d\rho^+}{dt}$ is dislocation generation.

Now we use these parameters. It is assumed that v is a power law function of the overstress:

$$v = c_1 \left(\frac{\tau - \tau_a}{T_1^*}\right)^M \tag{15}$$

Which c_1, T_1^* and M are constants (table 1). τ_a is described in power law strain hardening[11]:

$$\tau_a = \tau_{a0} \left[1 + \frac{\gamma^p}{\gamma_0}\right]^{1/n} \tag{16}$$

Which τ_{a0}, γ_0 and n are constants (see table1). ρ_m is mobile dislocation and the following form is adopted[11]:

$$\rho_m = \rho_{m0} \left[1 + (\frac{\alpha_b \gamma^p}{b\rho_{t0}}\right] \exp(-\alpha_t \alpha_b \gamma^p) \tag{17}$$

$\rho_{m0}, \rho_{t0,} \alpha_t, \alpha_b$ are constants. In acceding to irreversible thermodynamics [13]:

$$\frac{d\rho^+}{dt} = \frac{\tau \dot{\gamma^p}}{2E} \tag{18}$$

Which E=1/2Gb^2 is internal energy and G is shear modulus. τ described as phonon drag effect:

$$\tau = \tau_{a0}[1 - \exp(-\dot{\gamma}^p/\dot{\gamma}_0)] \tag{19}$$

With replacing eq. 15-18 to eq. 14, we have a relation with three unknown parameters τ, γ^p and $\dot{\gamma}^p$. Finally the problem of the propagation of steady plastic waves is governed by a system of nine equations [6, 7, 8, 10, 11, 12, 13, 14 and 19], the variables being $[\tau, \gamma^p, \dot{\gamma}^p, \sigma_1, \sigma_2, v_1, \varepsilon_1^p, \varepsilon_2^p, \lambda_1^p]$.

3. Results

Results are compared with [11]. Fig.1 shows stress vs longitudinal plastic stretch. Also other unknown variables can be investigated. The results have good agreement.

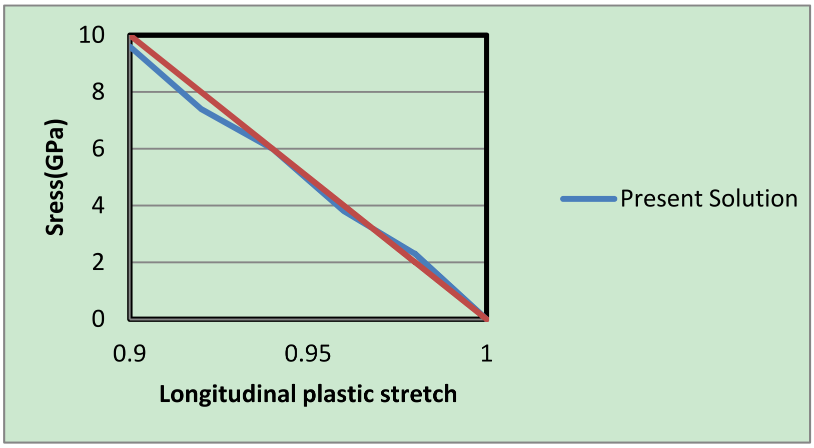

Fig.1: Stress vs Longitudinal plastic stretch.

Table 1: Material parameters for aluminium 6061-T6 at room temperature 25 °C and atmospheric pressure [11].

$\rho_0 = 2703$ kg/m^3	(mass density)
$c_L = 6368$ m/s	(longitudinal wave speed)
$c_S = 3197$ m/s	(shear wave speed)
Plastic characteristics provided by a quasistatic tensile test (hardening law	
$\tau_{a0} = 120$ MPa	(initial back stress)
$\gamma_0 = 0.52$	(reference strain)
$n = 1.55$	(hardening parameter)
Conjectured viscoplastic properties	
$M = 1/m = 1.78$	(m is the strain rate sensitivity)
$c_1 = 0.168$ ms^{-1}	
$T_1^* = 1.6$ MPa	
$b = 0.286 \times 10^{-9}$ m	(lattice parameter constant)
$N_{m0} = 0.818 \times 10^{13}$ m^{-2}	(initial mobile dislocation density)
$N_{t0} = 0.818 \times 10^{13}$ m^{-2}	(initial total dislocation density)
$\alpha_b = 3.5 \times 10^5$ m^{-1}	(breeding coefficient)
$\alpha_t = 0$	(trapping coefficient)

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