

SAFNAQ, An Algorithm for Transmission Power Allocation in MC-CDMA

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Abstract. A power allocation algorithm for MC-CDMA communication systems is proposed. There are many algorithms of power allocation in MC-CDMA, e.g., frequency domain power allocation algorithm (FDPA), time domain power allocation algorithm (TDPA) and eigenvector based power allocation algorithm (EBPA) etc., available in literature. The FDPA algorithm uniformly allocates power to some adaptively chosen strongest sub-carriers and all other deeply faded sub-carriers are shut off. In TDPA, the power adaptation is considered so that the desired signal strength at MRC receiver maintains at a fixed level. The EBPA algorithm allocates power in accordance with a power balance vector. This power balance vector is an eigenvector corresponding to the maximum eigenvalue of the modified projection matrix orthogonal to the interference signal subspace. The proposed algorithm, named SAFNAQ algorithm, is cohesion of FDPA and EBPA algorithms. It allocates transmission power to some adaptively chosen strongest sub-carriers not uniformly but in accordance with the power balance vector computed as in EBPA algorithm. Simulation results show that SAFNAQ algorithm outperforms both FDPA and EBPA algorithms as well as the no allocation algorithm.

Keywords: SAFNAQ, Multi-Carrier, CDMA, Power Allocation, Adaptive Systems, Rayleigh Fading.

1. Introduction

Multi-carrier code division multiple access (MC-CDMA) inherits the characteristics of both orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA). It appears most suitable for future 4G mobile wireless communications with high data rate. Its main advantages include multiple access capability, frequency diversity, robustness against fading, and ability to mitigate the inter symbol interference (ISI) [1]. Power allocation is the most important issue among many research issues of MC-CDMA that ensures the efficient use of power to maximize performance gain of the system. In multi-carrier communications, the channel fading for each user and at different sub-carriers is different. The multi-user water filling power allocation was derived on the idea of Cheng and Verdue [2]. They proposed that a user should use a sub-channel at which his sub-channel is best among all users. Many authors have proposed different schemes for resource allocation on the basis of channel state information (CSI) [3–5]. Zhu and Gunawan [5] proposed that the sub-carriers having channel gains higher than a given threshold should only be used and turn off the deeply faded sub-carriers and apply maximal ratio combining (MRC) at the receiver. This approach results in no transmission, when all sub-channel gains remain below the threshold level. However, it is acceptable for delay tolerable traffics only. Zhu and Bar-Ness [4] proposed an eigenvector based power allocation (EBPA) algorithm for MC-CDMA systems with projection matrix based receiver. Lee and Bar-Ness [3] proposed frequency domain power allocation (FDPA) and time domain power allocation (TDPA) schemes and combination of these both for uplink MC-CDMA transmissions with MRC receiver. However, cohesion of frequency domain power adaptation scheme [3] with optimum EBPA algorithm described in [4] has not yet been discussed for uplink MC-CDMA systems. This idea is implemented in form of SAFNAQ algorithm.

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The description of MC-CDMA system with projection matrix based receiver is presented in section 2. The SAFNAQ algorithm, with short introduction to FDPA and EBPA algorithms, is explained in section 3. Simulation results and conclusions are presented in sections 4 and 5 respectively.

2. System Description

A multiuser, synchronous, single cell MC-CDMA communication scenario with K active users communicating simultaneously and N number of sub-carriers is considered [4]. The power controlled MC-CDMA transmitter structure is depicted in Fig.1. The adaptation of Walsh Hadamard code is assumed into the system. The transmitter structure considered here, differs from the structure of conventional MC-CDMA transmitter by a small unit called power control unit, which is able to allocate different power to different sub-carriers and shut the deeply faded sub-carriers off. The low-pass equivalent transmitted signal can be expressed as

$$y_k(t) = \sum_{l=0}^{\infty} x_{k,l} \sum_{n=0}^{N-1} c_k[n] d_k[n] e^{\frac{j(2\pi n t)}{T_b}} p_{T_b}(t - lT_b), \quad (1)$$

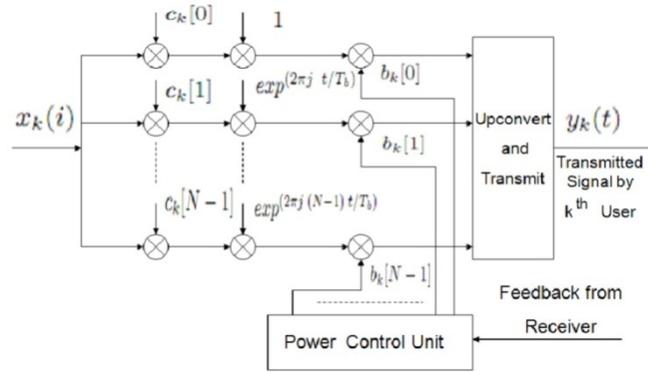


Fig. 1: Transmitter structure

Where k and l are the user and information data bit indexes respectively, and $x_{k,l}$ represents the l th information data bit of the k th user. $c_k[n]$ is the n th chip of spreading code and $d_k[n]$ is the n th element of the power balance vector corresponding to the k th user. The function $p_{T_b}(t)$ is the pulse shaping function with pulse width T_b . The power balance vector modifies the weights of sub-carrier signals to match the spectrum of $y_k(t)$ signal with the channel response. In a conventional MC-CDMA system power balance coefficients are defined as $d_k[n] = 1/\sqrt{N}$. The total average power $\bar{P} = \mathbf{d}_k^H \mathbf{d}_k$ is constrained and here \mathbf{d}_k is the power balance vector for k th user. The equivalent transmitted discrete signal corresponding to all users can be expressed as (we omit the bit index for compactness)

$$\mathbf{y} = \sum_{k=1}^K x_k \mathbf{C}_k \mathbf{d}_k, \quad (2)$$

where \mathbf{C}_k is a diagonal matrix and its diagonal elements $c_k[n]$ represent spreading code of k th user. Frequency selective Rayleigh fading channel with M received paths is considered as in [4]. Therefore the k th user's equivalent discrete received signal structure can be expressed as

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{C}_k \mathbf{d}_k, \quad (3)$$

where \mathbf{H}_k ($N \times N$) is a diagonal matrix with coefficients (sub-channel gains) of vector \mathbf{h}^k as its diagonal elements. The vector \mathbf{h}^k is the frequency response at sub-carriers defined in [4]. The receiver structure includes an extra unit than conventional MC-CDMA system and is depicted in Fig.2. This unit is used to feedback control information to each mobile station (MS). At the receiver, the discrete received signal can be expressed as

$$\mathbf{r} = \sum_{k=1}^K x_k \mathbf{r}_k \mathbf{V}, \quad (4)$$

where the vector \mathbf{V} represents the additive white Gaussian noise (AWGN) with co-variance matrix $\sigma_V^2 I_{N \times N}$. Project matrix based receiver structure is used to detect the desired signal. The vector \mathbf{r} is composed of desired signal $x_k \mathbf{r}_k$, interference signals $x_1 \mathbf{r}_1, \dots, x_{k-1} \mathbf{r}_{k-1}, x_{k+1} \mathbf{r}_{k+1}, \dots, x_k \mathbf{r}_k$ and noise signal \mathbf{V} . Project matrix for k th user is

$$\mathbf{P}_k = \mathbf{R}_k (\mathbf{R}_k^H \mathbf{R}_k)^{-1} \mathbf{R}_k^H, \quad (5)$$

where \mathbf{R}_k is the interference signal subspace consisting of vectors $\mathbf{r}_k, \dots, \mathbf{r}_{k-1}, \mathbf{r}_{k+1}, \dots, \mathbf{r}_k$. As CSI and spreading sequences of all of active users are available at receiver. Hence, \mathbf{P}_k is the project matrix to \mathbf{R}_k , and $\mathbf{I} - \mathbf{P}_k$ is the projection matrix to the subspace which is orthogonal to the $K - 1$ interfering signals. The estimated l th information data bit for k th user is determined by using Signum function $\text{sgn}(\cdot)$ [6], $x_k = \text{sgn}(\mathbf{r}_k^H (\mathbf{I} - \mathbf{P}_k) \mathbf{r}_k)$. For a particular channel condition, the overall system performance can be determined by finding signal to interference plus noise ratio (SINR) and then taking moving averages over different channel realizations as

$$\overline{Pe}(\text{error } \mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_k) = E [Q(\sqrt{\text{SINR}})] = E [Q(\sqrt{\frac{(\mathbf{r}_k^H (\mathbf{I} - \mathbf{P}_k) \mathbf{r}_k)^2}{\sigma_V^2 \mathbf{r}_k^H (\mathbf{I} - \mathbf{P}_k) \mathbf{r}_k + \mathbf{r}_k^H (\mathbf{I} - \mathbf{P}_k) \mathbf{R}_k \mathbf{R}_k^H (\mathbf{I} - \mathbf{P}_k) \mathbf{r}_k}})] \quad (6)$$

where the function $Q(\cdot)$ is Q-function [6].

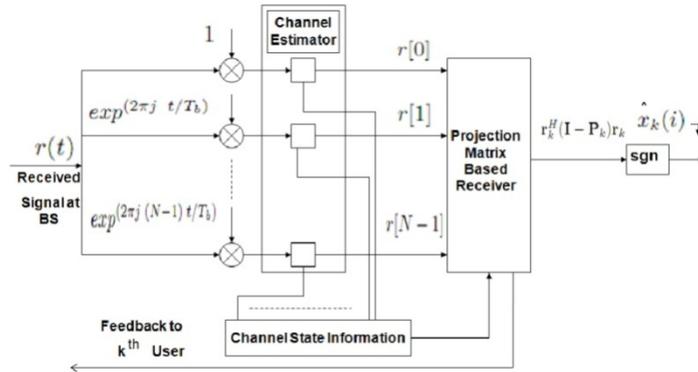


Fig. 2: Receiver structure

3. Power Allocation Using SAFNAQ Algorithm

Our objective is to optimize the power allocation in order to maximize the SINR under the constraints that the total transmit power should not exceed the maximum transmit power and each channel's SINR should not be less than a threshold value. Lee and Bar-Ness [3] proposed frequency domain power adaptation FDPA, time domain power adaptation TDPA schemes and combination of both of these schemes. In TDPA, they considered adapting the transmission power so that the desired signal strength at MRC receiver maintains at a fixed level. In FDPA, they proposed to allocate the transmission power uniformly over the sub-carriers out of sub-carriers which have the highest channel power gains. The elements of power balance vector can thus be expressed as

$$d_k^f[n] = \begin{cases} \frac{1}{\sqrt{N}}, & G_n^k > \varepsilon \\ 0, & G_n^k < \varepsilon \end{cases}, \quad (7)$$

where $G_n^k = (|\mathbf{h}_n^k|)^2$ is the n th sub-channel power gain for k th user. The constant ε is the optimum threshold, which dictates that how many sub-channels are in use at a time. \overline{N} is adjusted adaptively to obtain maximum SINR. The sub-carriers with gain below the threshold value will be shut off.

Zhu and Bar-Ness [4] considered the optimal power balance vector turned out to be the maximum eigenvalue eigenvector of modified projection matrix, represented as $\mathbf{C}_k^H \mathbf{H}_k^H (\mathbf{I} - \mathbf{P}_k) \mathbf{H}_k \mathbf{C}_k$, orthogonal to the interference signal subspace. This power balance vector is the eigenvector of the modified projection

matrix corresponding to the maximum eigenvalue (λ_{max}). This λ_{max} is found for each user and his power balance vector is replaced by the corresponding eigenvector. If it is already a suitable eigenvector, it is not changed.

It has already been proved that FDPA, TDPA and EBPA algorithms have better BER performance than conventional system [3, 4]. Fig.3 shows that FDPA algorithm is superior than EBPA algorithm upto about 7 dB, but EBPA algorithm performs better for higher values of SNR. FDPA requires smaller overhead bandwidth because it feeds back a single bit for each sub-carrier back to the transmitter. On the other hand EBPA algorithm feeds back real numbers i.e. the coefficients of the power balance vector. The EBPA algorithm maximizes the desired signal power in orthogonal direction to the interference signal subspace and converges more quickly in comparison to FDPA algorithm. The FDPA algorithm consumes a big fraction of the execution time in finding out the optimum value of \bar{N} .

We know that FDPA algorithm allocates power to \bar{N} strongest sub-carriers uniformly. But a question comes in mind that what if all of these N sub-carriers do not always have same channel gains, so should the transmission power be allocated uniformly to them? In EBPA algorithm, the transmission power is allocated in accordance with the maximum eigenvalue eigenvector of the modified projection matrix. No doubt, it gives better performance as compared to conventional system, but here again a question comes in mind. The question is that as if some of the sub-carriers are deeply faded, so shouldn't they be shut off? Thus these two questions gave the idea of optimum power allocation, which is implemented in the form of SAFNAQ algorithm.

As our main emphasis is on optimizing the power balance vector for better BER performance of the system and to reduce the feedback bandwidth overhead as well. A power allocation algorithm, named SAFNAQ algorithm, combines FDPA and EBPA algorithms for optimal allocation of available transmission power among sub-carriers. In SAFNAQ algorithm, we propose that total transmission power should be allocated to \bar{N} strongest sub-carriers not uniformly but they should adapt this power in accordance with the coefficients of \mathbf{d}_k^e . Shutting the deeply faded sub-carriers off, power allocated to these sub-carriers could be saved and allocated to the sub-carriers having better channel response.

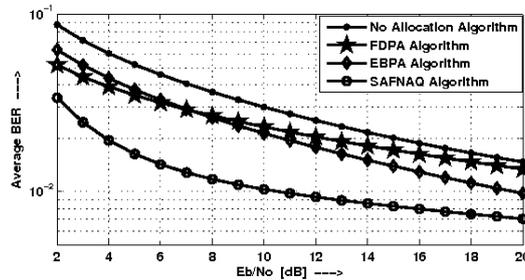


Fig. 3: Plot of average BER vs. SNR for $K=4$; $N=8$; $M=5$.

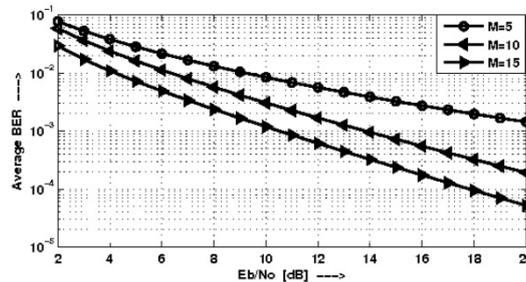


Fig. 4: Plot of average BER vs. SNR for $K=4$; $N=8$ for SAFNAQ algorithm.

Now the power balance vector for SAFNAQ algorithm can be expressed as

$$\mathbf{d}_k^s[n] = \begin{cases} \mathbf{d}_k^e[n], & G_n^k > \varepsilon \\ 0, & G_n^k < \varepsilon \end{cases} \quad (8)$$

and $\mathbf{r}_k, \mathbf{r}, \mathbf{P}_k, \text{SINR} \& \bar{P}_e$ change accordingly. Until it does not improve performance of the system, the same procedure is repeated for each user iteratively. The feedback overhead bandwidth in SAFNAQ algorithm is larger than or equal to FDPA algorithm and less than or equal to EBPA algorithm. Here the receiver needs to feedback N bits and \bar{N} real numbers. If we compare it with FDPA, EBPA and no allocation algorithms, it obtains the optimal overall BER performance by maximizing received signal energy.

4. Simulation Results

In this section performance of SAFNAQ algorithm is shown to compare with FDPA, EBPA and no allocation algorithms. It has also been shown that performance of SAFNAQ algorithm increases with the change of different parameters of the system. In our simulation, Walsh-Hadamard code is the spreading code with length of N . All users are assumed to suffer from independent and identical Rayleigh fading with M delay paths. It has been assumed that the channel coherence time is T_b and the variance of path gain for each path is the same, $\sigma_m^k = 1/M$.

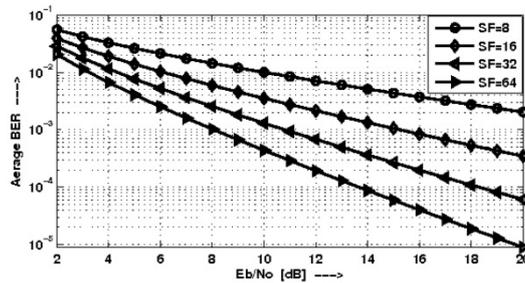


Fig. 5: Plot of average BER vs. SNR for $K=4; M=5$ for SAFNAQ algorithm.

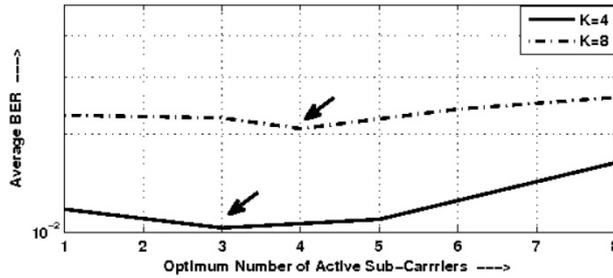


Fig. 6: Plot of average BER vs. \bar{N} for $N=8; M=5; Eb/No=10[\text{dB}]$ for SAFNAQ algorithm.

In Fig.3, the average BER performances of no allocation, FDPA, EBPA and SAFNAQ algorithms is shown. It can be seen that SAFNAQ algorithm is superior to FDPA and EBPA as well as no allocation algorithms for the considered range of SNR. Project matrix based receiver is used and each user transmits signal with power balance coefficient vectors $\mathbf{d}_k, \mathbf{d}_k^f, \mathbf{d}_k^e$ and \mathbf{d}_k^s respectively. Where the spreading gain N was set as 8, number of delay paths M as 5 and the number of active users K as 4. In Fig.4, the effect of correlation among sub-carriers on the performance of SAFNAQ algorithm with projection matrix based receiver is shown. We present BER performance with different number of resolvable delay paths $M=5, 10$ and 15. The more resolvable delay paths are available, the better performance can be achieved. We have to extend cyclic prefix length with increase in the number of resolvable delay paths. Fig.5 depicts the effect of the spreading factor (SF) on the BER performance when $K=4$ and $M=5$. The performance of the SAFNAQ algorithm increases with the increase of spreading factor. In Fig.6, BER curves for SAFNAQ algorithm with different number of active users K are shown and it can be noticed that there exists an optimal value of \bar{N} that minimizes the average BER. It indicates that for given system parameters, SAFNAQ algorithm can minimize the average BER by appropriately choosing the number of active sub-channels.

5. Conclusions

We considered in this paper, allocation of transmission power to the sub-carriers in MC-CDMA in a way to achieve better BER performance than both FDPA and EBPA algorithms. FDPA algorithm allocates power to \bar{N} strongest sub-carriers uniformly and EBPA algorithm allocates power in accordance with the maximum eigenvalue eigenvector of the modified projection matrix orthogonal to the interference signal subspace. While, SAFNAQ algorithm allocates power to \bar{N} strongest sub-carriers in accordance with the power balance vector found in the EBPA algorithm. It has been proved by simulation results that SAFNAQ algorithm has significant BER performance gain over both FDPA and EBPA algorithms. SAFNAQ may also reduce the requirement of overhead bandwidth than as it was in EBPA algorithm. Here the receiver have to send N on/off bits for each sub-carrier and $(1 \leq \bar{N} \leq N)$ real power balance coefficients for the sub-carriers that are not to be shut off. SAFNAQ algorithm feeds back N bits and \bar{N} real numbers instead of sending N real numbers because it will have lesser bandwidth overhead. Performance of SAFNAQ algorithm may also be increased by using some channel allocation technique and also by adapting power in time domain as done in TDPA algorithm.

6. References

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