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A Robust Beamformer against Large Pointing Error

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Abstract. In this paper, a low complexity beamformer with an iterative direction-of-arrival (DOA) estimation is proposed to alleviate performance degradation due to large pointing error. Numerical results demonstrate that, even in an interference-rich environment, the proposed beamformer can achieve comparable performance to that of the optimal one.

Keywords: beamformer, direction-of-arrival (DOA), pointing error, linearly constrained minimum variance (LCMV), and low complexity.

1. Introduction

It is well known that conventional adaptive beamformers are effective in suppressing strong interferers as long as the error in the steering vector due to pointing inaccuracy is small [1]. In the presence of pointing errors, these beamformers exhibit severe degradation in performance in that the output signal-to-interference-plus-noise ratio (SINR) drops dramatically. Remedies have been proposed to lessen the effect of desired signal cancellation [2]. In particular, the eigenspace beamformer (EIB) [3] uses the projection technique to remove the noise lying in the signal subspace and to boost the output SINR. In spite of the success in dealing with a moderate pointing error, this approach cannot completely block the interference and the residual interference that impairs system performance, especially for high the input signal-to-noise ratio (SNR) and/or weak interference. Furthermore, in an interference-rich environment, the EIB fails to achieve a reliable performance even in the case of small direction-of-arrival (DOA) estimation error. To mitigate the effect of large pointing errors, an iterative searching method [4] was considered to construct the correct constraint vector before beamforming. Its low convergence behavior because of a small tolerate value and/or a pointing error becomes crucial in practice. This approach, at worst, breaks down when the desired signal falls outside the main-beam region in the case of high input SNR and/or a large antenna array [5]. As an alternative, a two-stage scheme [6] incorporating the eigenspace technique was proposed to alleviate sensitivity to pointing errors. Unfortunately, this in turn results in large computational complexity.

The paper proposes a novel array processor with an iterative DOA estimation for enhanced robustness against pointing errors. Specifically, based on the phase-compensated subarray data, an iterative algorithm is derived to search for an accurate DOA of the desired source. The subarray weight vector is then determined in accordance with the linearly constrained minimum variance (LCMV) criterion, in which two symmetric directional constraints are employed to further improve the aggregate impacts due to the DOA estimation error in the first stage. Finally, a dimension recovery transformation is developed to construct the full-dimension weight vector acting on the received data. Numerical results indicate the efficacy of the proposed robust beamformer. The contribution of this work is threefold. First of all, an iterative scheme is proposed, in which only a slight increment in computational complexity load is required to promise an acceptable DOA estimate. Second, this study introduces the closed-form expression of the LCMV beamforming with two symmetric directional constraints for the virtual subarray, in which a negligible burden is induced. The third contribution of this work is that a full-dimension recovery transformation is developed to offer the maximum output SINR.

2. Array Data Mode

Scenario considered herein involves a single desired source and $K-1$ uncorrelated interfering sources, all assumed to be narrowband with the same center frequency. These sources are in the far field of an antenna array consisting of M identical elements spaced by half a wavelength. Adopting the complex envelope notation, the array data obtained at a certain sampling instant can be put in the $M\times 1$ vector form:

$$\mathbf{x}(n)=\sum_{k=1}^{K}s_k(n)\mathbf{a}(\theta_k)+\mathbf{n}(n). \tag{1}$$

The random scalars $s_k(n)$ for $k=1,2,\cdots,K$ represent the signals with power σ_k^2, in which, without loss generality, $s_1(n)$ is the desired signal. The $M\times 1$ vector $\mathbf{a}(\theta)=[1,e^{j\pi u},\cdots,e^{j(M-1)\pi u}]^T$ is the array steering vector, where $u=\sin\theta$ and θ is the physical angle measured with respect to the broadside of the array. Finally, the vector $\mathbf{n}(n)$ is additive white Gaussian noise with power $\sigma_k^2\mathbf{I}_M$.

In order to lessen the complexity in beamforming, the degree of freedom associated with all the tasks should be as small as possible. Reducing the dimension in processing suggests that the received data can be synthesized into $N=M/M_s$ contiguous non-overlapping subarrays of size M_s. The phase-compensated subarray data is given by

$$\tilde{\mathbf{x}}(n)=\sum_{l=1}^{N}e^{-j\pi(l-1)M_s u_s}\mathbf{S}_l\mathbf{x}(n), \tag{2}$$

where $u_s=\sin\theta_s$, with θ_s begin the look direction, and $S_l=[\mathbf{O}_{M_s\times(l-1)M_s},\mathbf{I}_{M_s},\mathbf{O}_{M_s\times(N-l)M_s}]$ is the selection matrix of the lth virtual subarray.

3. Proposed Low Complexity Beamformer

3.1. Iterative Searching Algorithm for DOA Estimation

It is well-known that an error in DOA estimate will cause the LCMV beamformer to suffer from desired signal cancellation and produce a low output SINR. As a remedy, a correct DOA estimate is necessary before combining the received data. One method of searching for an accurate DOA is to select the angle in the vicinity of the presumed DOA [7], which can achieve the minimum output power. Unfortunately, this is time wasting and the searching grid is not clear. As an alternative, an iterative searching algorithm based on the phased-compensated subarray data in (2) was proposed and summarized as following:

1. Construct the phase-compensated subarray data in (2).
2. Determine the initial refined DOA estimate from the candidate angle set $Q^{(0)}=\{\theta_s,\theta_s\pm B\}$, where B denotes the angular region of interest, in accordance with

$$\hat{\theta}^{(0)}=\min_{\theta\in Q^{(0)}}\mathbf{b}^H(\theta)\tilde{\mathbf{R}}^{-1}\mathbf{b}(\theta), \tag{3}$$

   where $\mathbf{b}(\theta)=\mathbf{S}_1\mathbf{a}(\theta)$ and $\tilde{\mathbf{R}}$ is the data correlation matrix associated with the phase-compensated virtual subarray data $\tilde{\mathbf{x}}(n)$ in (2).
3. In order to guarantee a more accurate DOA estimate, *Step 2* should be repeated and the DOA estimate $\hat{\theta}^{(i)}$ at the ith iteration is updated with the candidate angle set replaced by $Q^{(i)}=\{\hat{\theta}^{(i-1)},\hat{\theta}^{(i-1)}\pm 2^{-i}B\}$ until $2^{-i}B<\kappa$, where κ is the stop threshold. Note that there are a total of $J=\lfloor\log_2(\kappa^{-1}B)\rfloor$ iterations in this stage. Finally, the DOA estimate $\hat{\theta}_1$ is obtained by (3) with $Q^{(0)}$ replaced by $Q^{(J)}$.

It is noteworthy that, for the proposed scheme, the maximum tolerance error in the DOA estimate is $4(1-2^{-J})B$. In addition, since the major consideration in this stage is to remove desired signal cancellation caused by a large pointing error, the chosen stop threshold κ should be large enough to promise a low computational complexity but not too large to override the tolerance of the beamformer. To investigate the efficacy of the proposed searching scheme, an approximate formula for computational complexity, measured in terms of the number of complex multiplications, is examined. For batch processing, the major computational complexity in each step of the proposed algorithm involves the inversion of $\tilde{\mathbf{R}}$ in (3) about $O(M_s^3)$ and the multiplication $\mathbf{b}^H(\theta)\tilde{\mathbf{R}}^{-1}\mathbf{b}(\theta)$ equal to M_s^2. The overall complexity in this stage pertains to $O(M_s^3)+(2J+3)M_s^2$. For comparison, the conventional method with a uniform searching grid requires $O(M^3)+\lfloor 2\kappa^{-1}B \rfloor M^2$, which is much greater than that of the proposed scheme, especially for large M and/or small κ.

3.2. Optimal Beamforming

To further enhance robustness against the estimation error in the first stage, an LCMV beamformer should incorporate two symmetric directional constraints in the directions $\hat{\theta}_1 \pm \delta$, where $\delta = 2^{-J-1}B$. We have the weight vector of the subarrays given by [8]

$$\mathbf{w}_s = \tilde{\mathbf{R}}^{-1}\mathbf{C}\left(\mathbf{C}^H\tilde{\mathbf{R}}^{-1}\mathbf{C}\right)^{-1}\mathbf{C}^H\mathbf{b}(\hat{\theta}_1). \tag{4}$$

With sufficient degrees of freedom, the weight vector $\mathbf{w}_s$ effectively collects the desired signal and suppresses the interference. Unfortunately, working with a single subarray results in an array gain loss. A remedy would be to develop a combiner for each subarray, and then transform these combiner outputs by incorporating a weight vector $\tilde{\mathbf{w}}$. With the secondary combining, the overall processor is effectively a combiner that acts on the original full-dimension received data of size M. According to the maximum ratio combining technique [9], the weight vector of the secondary combiner is given by

$$\tilde{\mathbf{w}} = \tilde{\mathbf{a}}(\theta_1) = [1, e^{j\pi M_s u_1}, \cdots, e^{j\pi M_s (N-1) u_1}]^T. \tag{5}$$

Unfortunately, the true DOA θ_1 (u_1) in (5) is not available in practice. A feasible alternative is to replace θ_1 with $\hat{\theta}_1$. With $\tilde{\mathbf{w}}$ available, the overall full aperture weight vector $\mathbf{w}$ acting on $\mathbf{x}(n)$ can be derived by using the array output $y(n) = \mathbf{w}^H\mathbf{x}(n)$:

$$\mathbf{w} = [\mathbf{S}_1^T\mathbf{w}_s, \mathbf{S}_2^T\mathbf{w}_s, \cdots, \mathbf{S}_N^T\mathbf{w}_s]\tilde{\mathbf{a}}(\hat{\theta}_1) = diag\{\tilde{\mathbf{a}}(\hat{\theta}_1)\} \otimes \mathbf{w}_s \tag{6}$$

where $diag\{\cdot\}$ denotes a diagonal matrix and $\otimes$ is the Kronecker product of two matrices.

4. Computer Simulations

Computer simulations were conducted to ascertain the performance of the proposed robust low complexity beamformer. The array employed was a 32-element uniform linear array. All elements were assumed to be identical and omnidirectional. The scenario involved a desired source at $\theta_1 = -10^\circ$ with power $\sigma_1^2 = 1$, and the $K-1$ uncorrelated interferers uniformly distributed over the angular region $(20^\circ, 50^\circ)$ with the same power as the desired signal. The input SNR was set to be 10 dB and the subarray size is eight ($M_s = 8$). The angular region of interest was $B = 5^\circ$ and the stop threshold was $\kappa = 0.5^\circ$. For comparison, we also included the results obtained with the EIB [3], the EIB incorporating the refined DOA estimate obtained with the proposed searching algorithm (denoted as REIB), and the optimal beamformer, in which the correct look direction $\theta_s = \theta_1$ was used to compute the weight vector.

Figure 1 shows the curves of the output SINR versus the look direction θ_s varied from -7° to 7° (the null-to-null beamwidth of the broadside array is approximately 7.2° [10]). The results indicate that desired signal cancellation does not occur even with the desired source located out of the "main-beam". On the other hand, the EIB exhibits a significant degradation in SINR performance for a large pointing error ($>1^\circ$). Next, we examines the interference suppression capability with different values of K. The resulting output SINR is plotted in Fig. 2, which shows that the proposed scheme is likely to perform comparably to the optimal one even in an interference-rich environment $K > M_s$. On the contrary, the performance of the EIB significantly drops as the value of K increases. This is because, for a large value of K, the nonzero cross-correlation between the desired signal and interferers due to pointing errors increases the residual interference power and results in the EIB failing to produce a reliable performance. These are confirmed by the beam patterns shown in Fig. 3 obtained with $K = 5$ and 10, respectively. Clearly, in the case of $K = 5$, the proposed scheme can form a beam for collecting the desired signal and introduce significant attenuation for suppressing interference. This did not occur for the EIB. As regards to the case of $K = 10$, all beamformers are still able to impose sufficient attenuation on the interference directions to prevent performance breakdown. Unfortunately, the EIB puts a null to the direction of the desired source such that the desired signal cannot be retained, which in turn leads to a failure in beamforming. Finally, we investigates the convergence behavior by varying the data sample size (N_s) for computing the time-averaged version of the received data correlation matrix [10]. The results given in Fig. 4 demonstrate that the proposed beamformer with a similar performance as the optimal one converges in about 1000 data samples, which is about 0.22 dB away from the optimal case (infinite data samples, $N_s = \infty$). On the other hand, the other beamformers cannot collect the desired signal within a total of 5000 data samples due to both the effects of the pointing error and the finite samples.

5. Conclusion

In this paper, a low complexity beamformer with an iterative DOA estimation was proposed to combat large pointing errors. Compared with conventional beamformers, the proposed scheme exhibits better SINR performance and excellent robustness against pointing errors, especially for an interference-rich environment.

6. References

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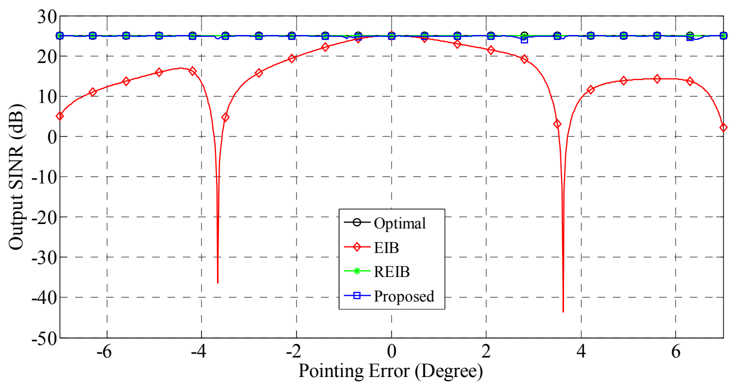

Fig. 1: Output SINR performance versus pointing error (θ_s). $K = 3$ and $N_s = \infty$.

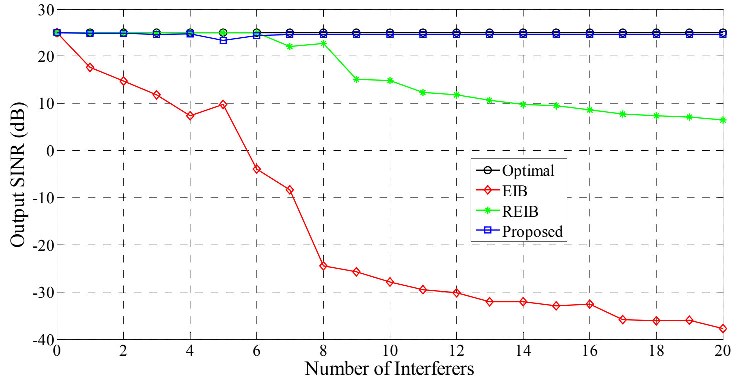

Fig. 2: Output SINR performance versus number of interferers (K). $\theta_s = 3.2^{\circ}$ and $N_s = \infty$.

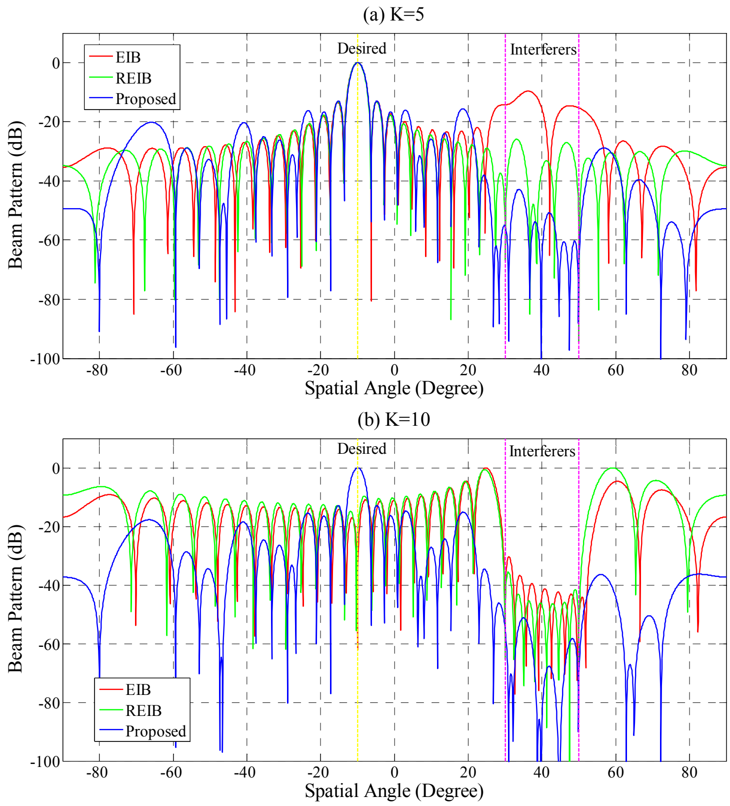

Fig. 3: Beam patterns obtained with $\theta_s = 3.2^{\circ}$ and $N_s = \infty$. (a) $K = 5$; (b) $K = 10$.

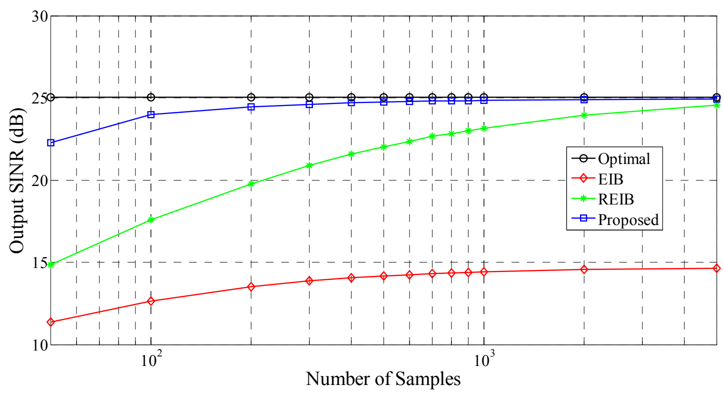

Fig. 4: Output SINR performance versus number of samples (N_s). $\theta_s = 3.2^{\circ}$ and $K = 3$.