

Parameter based Algorithm to Design Full State Controller for Second Order System

Engr. M. Bilal Shahid ¹, Engr. M. Abbas Abbasi ²

^{1,2} The Islamia University of Bahawalpur, Pakistan

Abstract: The work deals with the design and implementation of a second order system controller with a new approach based on the system parameters. The algorithm is simple and effective to use as compared with the previous traditional methods of controller designs. Due to the ease of computation of controller gains, the method can be implemented any control device including FPGAs, microcontroller or microprocessor and can be extended to higher order systems with a slight modifications. Matlab and Simulink have been used to test the functionality and response of the controller on a DC motor speed control system and it was verified that the method produces exactly the same results as the other traditional methods with less efforts and complexity.

Keywords: Matlab, Simulink, Control, Controller Design, Algorithm, Full State, DC Motor Control, Second Order Systems

1. Introduction

Controller design is one of most challenging and important step in any control system design problem. Various methods are employed to design the feedback gains for the full state controller including Direct method of comparing characteristic polynomials, canonical form method and Lyapunov method [1]. But these methods involve complex and somewhat tedious calculations and hence require more sophisticated hardware or software to compute the necessary control law. Due to the increased complexity for the higher order systems, it becomes extremely difficult to compute the large system matrices and matrix algebra arithmetic. We also require more memory to store such results.

The algorithm suggested here is very simple to use and is based on the coefficients of the systems matrices A, B, C and the desired pole locations. The control design becomes handy and the results produced are exactly the same for both the regulation and tracking scenarios.

A generalized second order system is considered and some new matrices and constants are introduced in the process of developing this algorithm. Then controller canonical technique is used to develop the control law. First characteristic polynomial of the systems is written and it is proved that the sum of the Eigen values for any order system is equal to the sum of the diagonal entries as in the case of a Jordan Matrix. Then it is shown that the product of the system poles is equal to the determinant of the matrix A.

The desired pole locations are taken into account and their sum and product are defined as the new parameters. It is assumed in the beginning that the system is controllable and hence controllability matrix M is calculated and it manipulated with the Toeplitz Matrix to form P matrix as is done in the canonical form method. The difference between polynomial equations' coefficients is considered as intermediate gain and at the end final control law is designed using P matrix and the intermediate gain matrix. The gain matrix is dissolved into gain formulae and these formulae are based purely on system parameters and desired pole location which makes this method simple and efficient.

Using the recommended control law, the controller is developed in Simulink. Masked system is employed to show the simplicity of the algorithm. Then the controller is implemented on a second order DC motor speed

control system. At the end it is shown that the same technique can be used to design the full state and reduced state estimators and compensators to make the control more effective.

2. Algorithm Development

2.1 System Description

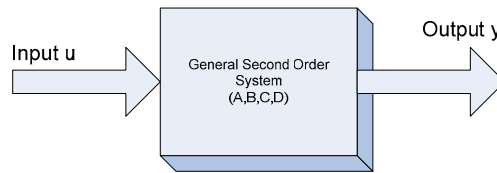


Fig. 1 General Second Order System in State Space

Consider any second order system which is represented in the form of state space form as follows [1] and [2]:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (1)$$

Where A, B, C and D are system matrices and x represents the state variables and y the output of the system. For simplicity, let $D = 0$. Then the generalized second order system can be written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u, \quad y = [c_1 \quad c_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (2)$$

Let λ_{a1} and λ_{a2} be the actual poles of the above system and λ_{d1} , λ_{d2} be the desired new locations of the poles. These new locations will correspond to new feedback control gains K_1 and K_2 .

Let:

$$A_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} \quad \& \quad A_2 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} \quad (3) \quad \begin{aligned} S_{d\lambda} &= \lambda_{d1} + \lambda_{d2} \\ S_{a\lambda} &= \lambda_{a1} + \lambda_{a2} = a_{11} + a_{22} \end{aligned} \quad (4)$$

$$P_{d\lambda} = \lambda_{d1}\lambda_{d2}$$

$$P_{a\lambda} = \lambda_{a1}\lambda_{a2} = |A| \quad (5)$$

2.2 Method Development

Consider the system block diagram shown in Fig. 2. The state space equations of the systems can also be written as [2] & [3]:

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + b_1u \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + b_2u \\ y &= c_1x_1 + c_2x_2 \end{aligned} \quad (6)$$

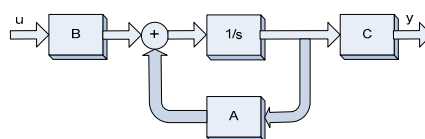


Fig. 2 The general block diagram of the system

Let us introduce now a feedback control matrix K such that [4]:

$$u = r - Kx = r - [K_1 \quad K_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (7)$$

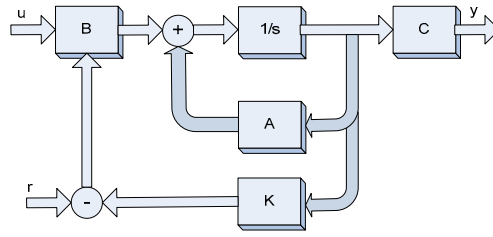


Fig. 3 System block diagram showing feedback control to place the poles at new locations

We will use canonical method to compute the gains. For this purpose we need characteristic polynomial $\Delta(s)$ and the desired polynomial $\Delta_f(s)$.

$$\Delta(s) = |sI - A| = s^2 - (a_{11} + a_{22})s + (a_{11}a_{22} - a_{12}a_{21}) \Rightarrow \Delta(s) = s^2 - S_{d\lambda}s + |A| = s^2 - S_{d\lambda}s + P_{a\lambda} \quad (8)$$

And using the desired pole locations we can form the desired polynomial as:

$$\Delta_f(s) = (s - \lambda_{d1})(s - \lambda_{d2}) = s^2 - (\lambda_{d1} + \lambda_{d2})s + \lambda_{d1}\lambda_{d2} \Rightarrow \Delta_f(s) = s^2 - S_{d\lambda}s + P_{d\lambda} \quad (9)$$

Using the coefficients of (8) and (9) we can compute the intermediate gain \bar{K} [3] & [4]:

$$\bar{K} = [\bar{\alpha}_2 - \alpha_2 \quad \bar{\alpha}_1 - \alpha_1] = [-S_{d\lambda} + S_{a\lambda} \quad P_{d\lambda} - P_{a\lambda}] \quad (10)$$

And the final feedback control gain K will be:

$$K = [K_1 \quad K_2] = \bar{K}P \quad (11) \quad \text{Where } P \text{ is the transformation matrix given by: } P = (MW)^{-1} \quad (12)$$

M is the controllability matrix of the system and W is the Toeplitz matrix and they can be calculated from the following relations [4] & [8]: $M = [B \quad AB] = \begin{bmatrix} b_1 & a_{11}b_1 + a_{12}b_2 \\ b_2 & a_{21}b_1 + a_{22}b_2 \end{bmatrix}$ $W = \begin{bmatrix} 1 & \alpha_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -(a_{11} + a_{22}) \\ 0 & 1 \end{bmatrix}$ (13) (14)

Putting (13) and (14) in (12) and simplifying we can show that:

$$MW = \begin{bmatrix} b_1 & -A_1 \\ b_2 & -A_2 \end{bmatrix}, P = \frac{1}{A_1b_2 - A_2b_1} \begin{bmatrix} -A_2 & A_1 \\ -b_2 & b_1 \end{bmatrix} \quad (15) \quad \text{And finally}$$

$$K_1 = \frac{-A_2(S_{d\lambda} - S_{a\lambda}) - b_2(P_{d\lambda} - P_{a\lambda})}{A_1b_2 - A_2b_1}, K_2 = \frac{A_1(S_{d\lambda} - S_{a\lambda}) + b_1(P_{d\lambda} - P_{a\lambda})}{A_1b_2 - A_2b_1} \quad (16)$$

The above results represent the final feedback controller gains in terms of actual system pole locations and desired pole locations and system matrices A and B . The calculation of the gains is simple and straightforward and involves no matrix algebra whatsoever. This situation is ideal for the implementation of controller on FPGAs and Digital controllers.

3. Implementation

3.1 DC Motor Speed Control

The algorithm is implemented in Simulink for a second order DC motor speed control problem. The equivalent diagram for the DC motor is shown in fig. 4.

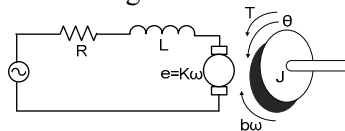


Fig. 4 Equivalent diagram for DC motor speed control system

$$\begin{bmatrix} \dot{\omega} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u \quad \text{The state space model of the motor can be written as [3] & [8](17)}$$

$$y = [1 \quad 0] \begin{bmatrix} \omega \\ i \end{bmatrix}$$

ω : Motor Speed (rad / s) i : Motor Current (A), b : Damping Ratio (Nms) $K(K_e = K_t)$: emf Constant (Nm / A)

J : Moment of Inertia ($\text{Kg.m}^2 / \text{s}^2$) R : Resistance (Ω), L : Inductance (H)

Comparing the DC motor speed control system with that of (2), we have:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \quad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \quad [c_1 \ c_2] = [1 \ 0] \quad (18)$$

Using above values and desired pole locations $\lambda_{d1}, \lambda_{d2}$ we can easily calculate $A_1, A_2, S_{d\lambda}, S_{d\lambda}, P_{d\lambda}$ to compute the values of K_1 and K_2 .

3.2 Simulink Controller Implementation

The controller gains are calculated using simulink's masked subsystem block and the control algorithm is implemented for the DC motor speed control system. The simulation diagrams are shown below:

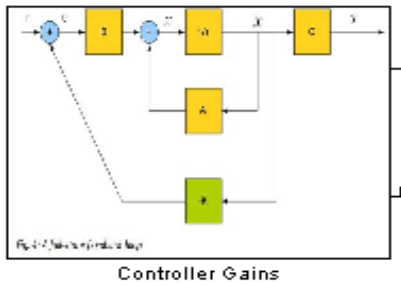


Fig. 5 Masked subsystem to implement the controller in Simulink

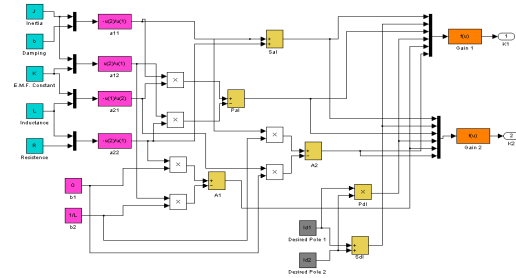


Fig. 6 Simulink Controller diagram to calculate feedback control gains



Fig. 7 Message box, prompting user to enter the different

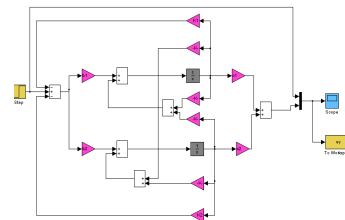


Fig. 8 Simulation diagram of the complete DC motor speed operating parameters and desired pole locations control system along with the feedback controller for the calculation of the feedback gains

The masked subsystem of the Simulink is an efficient way to quickly check the behavior of the controller under different DC motor parameters [7]. When user double clicks the controller block, the message box shown in fig. 7 appears which prompts the user to enter different simulation parameters and desired pole locations of the system. This makes it convenient to observe the response for different operating condition.

3.3 DC Motor Speed Control System Implementation in Simulink

The simulation diagram of the complete system is shown in fig. 8. The feedback control gains are taken from the above diagram and are used here. The advantage of this scheme is that we need not design the controller every time when there is a change in any system parameter.

4. Simulation and Results

The simulation was performed for a DC motor whose different parameters were taken as given Table 1:

Table 1 DC Motor Simulation Parameters and Desired Pole Locations

Symbol	Parameter
<i>Moment of Inertia (J)</i>	0.01 (Kg.m ² /s ²)
<i>Damping Ratio (b)</i>	0.1 (Nms)
<i>Emf Constant (K)</i>	0.01 (Nm/A)
<i>Resistance (R)</i>	1.3 Ω
<i>Inductance (L)</i>	300 mH
<i>Desired pole Locations</i>	Tested for different pole locations

4.1 Results

The actual pole locations were calculated as -9.99 and -4.33 which lie in the left half plane. The simulation was run for various desired locations of the poles and the results obtained are shown below:

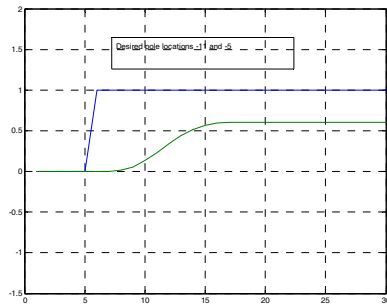


Fig. 9 Simulation results for desired pole locations of -11 and -5 farther in left half plane from the actual pole locations -4.33 and -9.99 . Using a forward gain we can exactly track the input. The system remains stable in this region

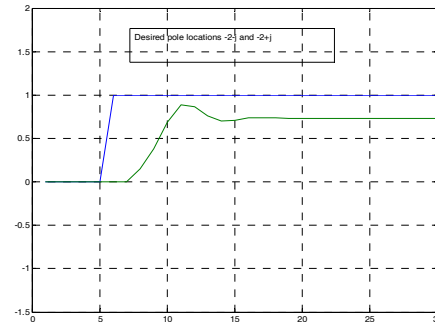


Fig. 9 Simulation results for desired pole locations of $-2-j$ and $-2+j$ in left half plane. Using a forward gain we can exactly track the input. The system remains stable in this region. The system shows an overshoot in this scenario

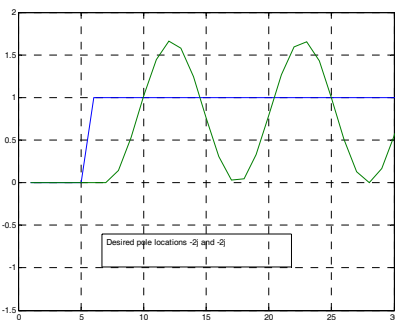


Fig. 10 Simulation results for desired pole locations of $-2j$ and $-2j$ on the imaginary axis. The controller response is oscillatory for the given conditions and is not controllable

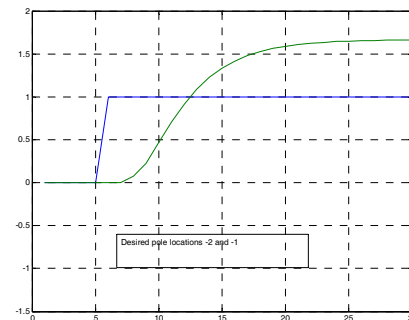


Fig. 11 Simulation results for desired pole locations of -2 and -1 in the left half plane right to the actual pole locations. When the desired poles lie to the right of the actual poles the response tends to become unstable one.

If we move the poles to the right and also include the oscillation frequency component in the pole location, the response generates some overshoot and then settles down to the steady state. The steady state can be equalized to the input steady state level if we include some forward gain in the output loop. Moving poles farther to the right will make the control system unstable. If poles are moved to imaginary axis, oscillatory behavior can be observed.

5. Conclusion

The algorithm is very simple and straightforward and involves no matrix algebra operations. It can easily be extended to higher order systems to design not only the feedback controllers but also the state estimators (observers) and compensators. Calculation of the feedback gains can easily be implemented using any hardware device such as FPGAs, CPLDs or digital controller ICs due to the algebraic simplicity of the proposed method.

6. References

- [1] Robert L. Williams, Douglas A. Lawrence "Linear state space control systems" John Wiley and Sons, 2007
- [2] Katsuhiko Ogata "State space analysis of control systems" Prentice hall 1967
- [3] Stanley M. Shinnars "Modern control system theory and design" wiley-IEEE, 1998 second edition.
- [4] Chi-Tsong chen "Linear systems theory and design" New york oxford university press 1999
- [5] Elbert Hendricks, O.E. Jannerup, Paul Haase sorensen "Linear system control: deterministic and stochastic methods" 2008 Springer-Verlag Berlin Heidelberg
- [6] K.Warwick "An introduction to control systems" Advanced series in electrical and computer engineering-vol.8, second edition 1989.
- [7] Alberto, Roberto setola, Francesco Vasca "Using Matlab, Simulink, and control system toolbox: a practical approach " Prentic Hall, 1996.
- [8] Shlomo Engelberg " A mathematical introduction to control theory " Imperial college press, 2005.