2011 International Conference on Modeling, Simulation and Control
IPCSIT vol.10 (2011) © (2011) IACSIT Press, Singapore

Linearized State Space Modeling of PEMFC for Control Applications

Engr. M. Abbas Abbasi [1], Engr. M. Bilal Shahid [2]

[1,2] The Islamia University of Bahawalpur, Pakistan

Abstract: This paper focuses on the linearization study of the non-linear dynamic models of Polymer Electrolyte Membrane Fuel Cells (PEMFC). Matlab and Control Systems Toolbox have been employed to study the model. The FC stack voltage is dependent upon the current drawn from the cell, the change in partial pressures and the change in the amount of inlet flow of water vapor, hydrogen and air. A sudden change in the state space model inputs is applied to the developed model and the system's step response is studied. The partial pressures of H2, O2 and H2O are selected as the state variables of the system.

Keywords: Fuel Cell, Modeling, Matlab, Control, PEMFC, State Space, Linearization

1. Introduction

Fuel cells convert chemical energy to electrical energy based on various operating principles without producing the byproducts typically associated with conventional power generation methods. They are environment friendly and have a great potential for higher efficiencies. There are numerous types of fuel cells including Proton Exchange/Polymer Electrolyte Membrane Fuel Cells (PEMFC), Alkaline Fuel Cells (AFC), Direct Methanol Fuel Cells (DMFC) etc. differing from one another by the type of fuel they use and the operating temperature [1].

PEMFC use hydrogen as fuel and operate at low temperatures. The energy conversion phenomenon associated with fuel cells is very complex and needs thermodynamic as well as chemical reaction equations to be expressed exactly.

Extensive work is being done in this field to model the fuel cells in different perspectives and under different operating conditions. Although numerous state space and other PEMFC models are available, however they are too complex to be understood and be utilized for effective control applications. The fuel cell dynamics are nonlinear in nature and a large signal state space model is very difficult to establish. However it is very easy to linearize these models for small changes in the state variables [2]. Such approach has been adopted and a small signal linearized model is developed for a general PEMFC. Later this model is converted to time domain by taking the inverse Laplace transform of the General State Space Matrix G(s).The model is attractive from the control point of view and can easily be extended for other applications.

1. System description and model development

1.1. PEMFC Operation

Fig. 1 shows a simplified schematic of the operation of PEMFC. The figure shows that pressurized hydrogen (fuel tank) and oxygen taken from the air are fed to the fuel cell from anode and cathode sides respectively. Reformed methane and other fuels like natural gas can also be fed instead of hydrogen. The partial pressures of both hydrogen and oxygen need to be regulated. The quality of fuel as well as air is also of crucial importance. Higher stack voltage can be produced by connecting the various cells in series. In the presence of the electrolyte catalyst, chemical reaction takes place between hydrogen and oxygen and as a result water and electricity is produced which can be summarized as [3]:

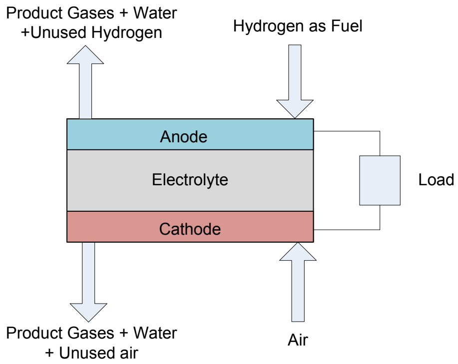

Fig. 1 PEMFC operation

$$Anode: \quad H_2 \rightarrow 2H^+ + 2e^-$$
$$Cathode: \quad \frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O + Electrical\ Energy + Heat \qquad (1)$$

1.2. Stack Voltage

The voltage of the fuel cell can be calculated using the relation [4]:

$$V_{FC} = V_{rev} - V_{losses} \qquad (2) \qquad V_{losses} = V_{act} + V_{ohm} + V_{con} \qquad (3)$$

Where,

V_{rev}: Thermodynamic or reversible voltage. Also known as Nernst Potential

V_{FC}: Fuel Cell Stack voltage V_{losses}: Voltage losses in the fuel cell V_{act}: Activation losses due to the rate of chemical reactions on electrodes surface V_{ohm}: Electrical resistance by the electrolyte to the flow of electrons V_{con}: Voltage loss due to concentration reduction

The above voltages can be calculated from the following formulae:

$$V_{rev} = N\left[V_{oc} + \frac{RT}{2F}\ln\left(\frac{p_{H_2}\sqrt{p_{O_2}}}{p_{H_2O\_C}}\right)\right] \qquad (4) \qquad V_{act} = \frac{NRT}{2\alpha F}\ln\left(\frac{J+J_n}{J_o}\right)$$

(5)

$$V_{ohm} = NiR_{FC} \qquad (6) \qquad V_{con} = Nce^{di}$$

(7)

Where,

N: Stack cells R: Universal gas constant T: Cell temperature F: Faraday's Constant

pH_2, pO_2, p_{H2O_C}: partial pressures of hydrogen, oxygen and water vapor from cathode side

J: Load current density J_o: exchange current density J_n: internal current density R_{FC}: Cell area specific resistance c & d: constants

The equivalent circuit model [5] of the fuel cell voltages is shown in fig. 2. And fuel cell V-I characteristic curve [6] is shown in fig. 3.

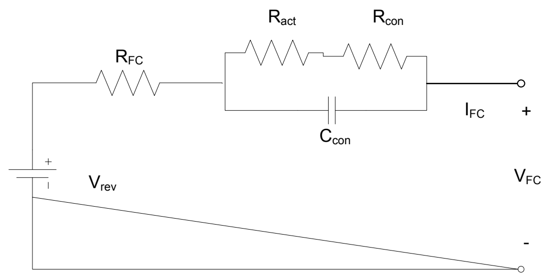

Fig. 2 Equivalent circuit model of the fuel cell voltages

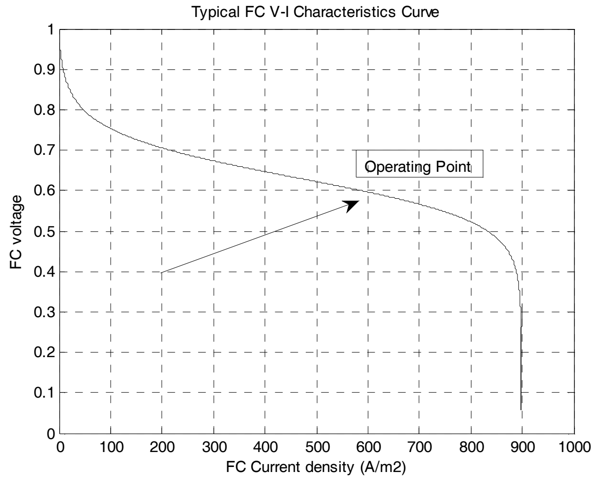

Fig. 3 Typical FC V-I curve

1.3. Model development

To develop the small signal state space model, the perturbations in hydrogen, oxygen and water vapor pressures ($\delta p_{H_2}, \delta p_{O_2}, \delta p_{H_2O\_C}$) were selected as the state variables of the system and small variations in the inlet flow rates of fuel injected (δF_{H_2}) , air (δF_{O_2}) , water ($\delta F_{H_2O\_C}$) and the load current (δi_L) were selected as the inputs. The fuel cell voltage change (δV_{FC}) was selected as the only output of the linearized system.

The linearized state equations can be written as:

$$\dot{\delta x} = A\delta x + B\delta u, \quad \delta y = C\delta x + D\delta u, \quad \dot{\delta x} = \frac{d}{dx}(\delta x), \quad \delta x = \begin{bmatrix} \delta p_{H_2} \\ \delta p_{O_2} \\ \delta p_{H_2O\_C} \end{bmatrix} \delta u = \begin{bmatrix} \delta F_{H_2} \\ \delta F_{O_2} \\ \delta F_{H_2O\_C} \\ \delta i_L \end{bmatrix}, \quad \delta y = \delta V_{FC}$$

(8)

While the matrices are given as [7]:

$$A = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}, \quad C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \end{bmatrix}, B = \begin{bmatrix} B_{11} & 0 & 0 & B_{14} \\ 0 & B_{22} & B_{23} & B_{24} \\ 0 & B_{32} & B_{33} & B_{34} \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & D_{14} \end{bmatrix}$$

(9)

And the matrices' entries are defined below [8]:

$$A_{11} = -\frac{RT\left(F_{an} - \frac{Ni}{2F}\right)}{k_{an}}, A_{22} = -\frac{RT\left(F_{cath} - \frac{Ni}{4F}\right)}{k_{cath}}, A_{33} = -\frac{RT\left(F_{cath} + \frac{Ni}{2F}\right)}{k_{cath}}, B_{11} = RT\left(\frac{1}{v_{an}} - \frac{p_{H_2}}{k_{an}X_{H_2}}\right)$$

$$B_{14} = -\frac{Na_{FC}\Delta p_{H_2}}{2Fk_{an}}, B_{22} = RT\left(\frac{1}{v_{cath}} - \frac{p_{O_2}}{k_{cath}(X_{O_2} + X_{H_2O\_C})}\right), B_{23} = RT\left(-\frac{p_{O_2}}{k_{cath}(X_{O_2} + X_{H_2O\_C})}\right)$$

$$B_{24} = -\frac{Na_{FC}\Delta p_{O_2}}{4Fk_{cath}}, B_{32} = RT\left(-\frac{p_{H_2O\_C}}{k_{cath}(X_{O_2} + X_{H_2O\_C})}\right), B_{33} = RT\left(\frac{1}{v_{cath}} - \frac{p_{H_2O\_C}}{k_{cath}(X_{O_2} + X_{H_2O\_C})}\right)$$

$$B_{34}=\frac{Na_{FC}\Delta p_{H_2O\_C}}{2Fk_{cath}},C_{11}=\frac{NRT}{2Fp_{H_2}},C_{12}=\frac{NRT}{4Fp_{O_2}},C_{13}=-\frac{NRT}{2Fp_{H_2O\_C}}$$

and $D_{14}=-N\left(R_{FC}+\frac{a}{J+J_n}+\frac{b}{J_1-J-J_n}\right)$

(10)

The entries defined in (10) involve various fuel cell parameters summarized in table 1.

Table 1: Fuel Cell Parameters used in the model

Symbol	Parameter	Symbol	Paramter
F_{an}	Total inlet flow into the anode (L/min)	X_{H2}	Anode inlet hydrogen mole fraction
i	Operating current (A)	X_{O2}	Cathode inlet oxygen mole fraction
F_{cath}	Total inlet flow into the cathode (L/min)	X_{H2O_C}	Cathode inlet water vapor mole fraction
k_{an}	$k_{an}=v_{an}p_o$ Anode energy constant . p_o is operating pressure of fuel cell (J)	a_{FC}	Fuel Cell active area (cm^2)
k_{cath}	$k_{cath}=v_{cath}p_o$ cathode energy constant . p_o is operating pressure of fuel cell (J)	Δp_{H2}	Difference between fuel pressure and fuel cell operating pressure (p_o)
v_{an}	Volume of anode (cm^3)	Δp_{O2}	Difference between oxygen pressure and fuel cell operating pressure (p_o)
v_{cath}	Volume of cathode (cm^3)	Δp_{H2O_C}	

Matlab's Symbolic Math toolbox was used to generate the transfer matrix $G(s)=\Delta Y(s)/\Delta U(s)$ which equals:

$$G(s)=C(sI-A)^{-1}B+D.$$

By using this approach, the following four transfer functions were obtained.

$$\frac{\Delta Y(s)}{\Delta U_1(s)}=\frac{B_{11}C_{11}}{s-A_{11}}\Rightarrow B_{11}C_{11}e^{A_{11}t},\frac{\Delta Y(s)}{\Delta U_2(s)}=\frac{B_{22}C_{12}}{s-A_{22}}+\frac{B_{32}C_{13}}{s-A_{33}}\Rightarrow B_{22}C_{12}e^{A_{22}t}+B_{32}C_{13}e^{A_{33}t}$$

$$\frac{\Delta Y(s)}{\Delta U_3(s)}=\frac{B_{23}C_{12}}{s-A_{22}}+\frac{B_{33}C_{13}}{s-A_{33}}\Rightarrow B_{23}C_{12}e^{A_{22}t}+B_{33}C_{13}e^{A_{33}t}$$

$$\frac{\Delta Y(s)}{\Delta U_4(s)}=D_{14}+\frac{B_{14}C_{11}}{s-A_{11}}+\frac{B_{24}C_{12}}{s-A_{33}}+\frac{B_{34}C_{13}}{s-A_{33}}\quad\Rightarrow D_{14}\delta(t)+B_{14}C_{11}e^{A_{11}t}B_{24}C_{12}e^{A_{22}t}+B_{34}C_{13}e^{A_{33}t}$$

These transfer functions basically predict the change in the fuel cell output voltage when any of the controlling parameter is varied while other perturbations are kept zero.

2. Simulation And Results

Matlab and Control systems toolbox were used to investigate the proposed model. The parameter values used in the study are enlisted in Table 2. All the parameters were defined in the Matlab and matrices A, B, C and D were formed. Using these matrices the behavior of the model was studied under different input scenarios.

Table 2: Fuel Cell Parameter Values used in Simulation

Parameter	Value		
N	6	*Cathode Volume*	19.48 cm^3
F	96485 C/mol	*Operating Pressure*	1 atm.
X_{H2}	0.91	*Operating Temperature*	72˚C
X_{O2}	0.189	R_{FC}	0.00032 Ω
Hydrogen inlet Flow	4.2 L/min	*a(Tafel Line Constant)*	0.07 V
Air inlet flow	13.5 L/min	*b (Concentration losses constant)*	0.06 V
Cell Active area (a_{FC})	198 cm^2	*Anode Volume*	10.391 cm^3
Load Current	45 A		

Fig. 4 shows that when inlet fuel is perturbed it will cause a similar change in the FC stack voltage. Fig. 5 shows the response to perturbations in air flow. But these perturbations do not affect the output as much as the fuel inelt flow perturbations.

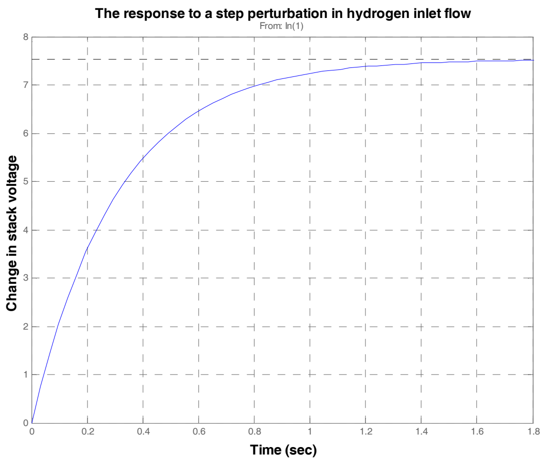

Fig. 4 The response to a perturbation in fuel inlet flow

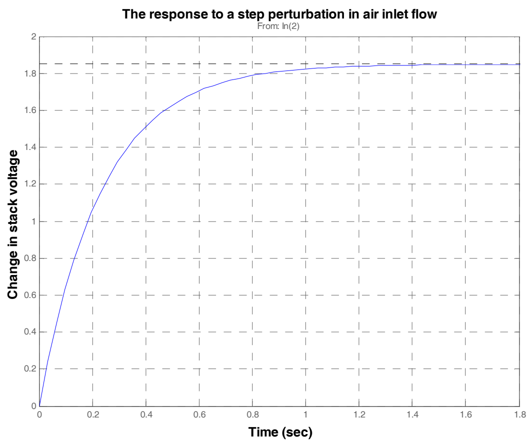

Fig. 5 The response to a perturbation in air inlet flow

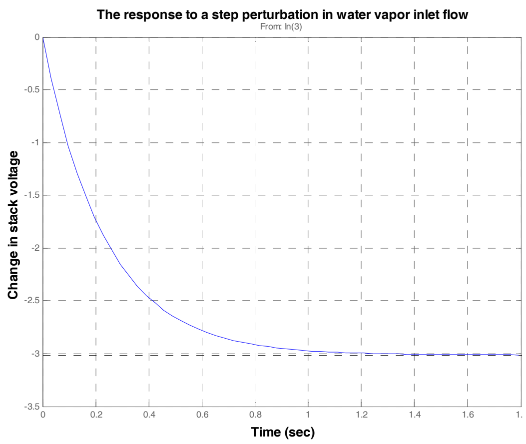

Fig. 6 Step response to the variations in water vapor flow

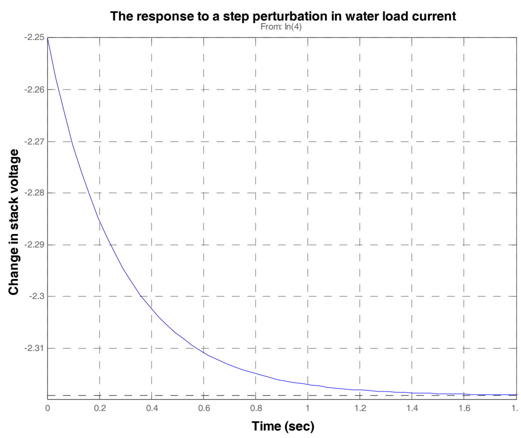

Fig. 7 The response to variations in the load current

Similarly fig. 6 and fig 7 show the PEMFC voltage behavior with respect the perturbations in water vapor inlet flow and load variations. The plot shown in fig. 7 is the most important one as it predicts the stack output voltage changes when load of the PEMFC is varied. The transient response is of prime importance in the figures above. The perturbations were settled down in different times. For the given conditions settling times for the four input disturbances were noted as 1.21s, 0.92s, 0.915s and 1.13s.

3. Conclusion

The model is suitable for the control applications and the analysis can easily be extended to the combined effect of multiple perturbations occurring in the fuel cell at the same time. The study shows the importance of the simplicity of the model and its usefulness in the control scenario. The model helps for the design and optimization strategies of the control schemes. It can also be used in the integrated systems and to predict the system's response under different frameworks. The simulations also indicate the accurate and logical behavior of the PEMFC for the step inputs. It can also be tested under various input perturbations. The similar techniques can also be applied to analyze other types of fuel cells. If we compare the model suggested here with the previous work, then we will find that the proposed model is simpler and effective for small signal applications. Some of the models are suggested in [5] and [8]. These linearized models involve lot of parameters and take into account the thermodynamic processes in the fuel cells which create complexities for common control applications. The suggested model is not only valid for linear operation but can also be extended to multivariable control scenarios.

4. Refernces

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