2011 International Conference on Modeling, Simulation and Control
IPCSIT vol.10 (2011) © (2011) IACSIT Press, Singapore

Finite Element Modeling of a Multilayered Sandwich Beam with Viscoelastic Core for Vibration Analysis

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Abstract: This work proposes a finite element for modeling a generalized multi-layered symmetric sandwich beam with alternate elastic and viscoelastic layers. The detailed derivation of the element mass and stiffness matrices have been presented. Numerical results have been presented for three, five and seven layered. The proposed element can be used for vibration analysis of sandwich beams having any number of layers.

Key Words: Finite element, Multilayer, Sandwich beam and viscoelastic core.

1. Introduction

Vibration control of machines and structures incorporating viscoelastic materials in suitable arrangement is an important aspect of investigation [1]. The use of viscoelastic layers constrained between elastic layers is known to be effective for damping of flexural vibrations of structures, over a wide frequency range. Multilayered cantilever sandwich beam like structures can be used in aircraft structures and other applications, such as robot arms for effective vibration control.

DiTaranto and Balsingame [2] obtained composite loss factor for selected laminated beams. Mead and Markus [3] studied the forced vibration of a three layer damped sandwich beam with arbitrary boundary conditions. Rao [4] calculated the frequency parameters and loss factors of sandwich beams under various boundary conditions and presented them in the form of equations and graphs. Banerjee [5] carried out free vibration analysis of three layered symmetric sandwich beams using dynamic stiffness method.

The theory of flexural vibration of symmetric multilayered beams was analysed by Agbasiere and Grootenhuis [6]. Asnani and Nakra [7] investigated the flexural vibration of multilayered unsymmetrical beams. Asnani and Nakra in their later work [8] explored the damping effectiveness during flexural vibration of multilayered beams with number of layers up to 15, with simply supported end conditions. Vaswani, et al. [9] obtained resonant frequency and system loss factor for general multilayered curved beams.

The purpose of the present work is to study the parametric instability of a multilayered cantilever sandwich beam subjected to end periodic axial load. Equation of motion for a general 2n+1 layered beam is derived using finite element method in conjunction with Hamilton's principle.

2. Formulation of the Problem

A 2n+1 layer sandwich beam, incorporating viscoelastic damping material is shown in figure(1). There are n number of viscoelastic layers and n+1 numbers of elastic layers. A layer of viscoelastic material separates two adjacent stiff elastic material layers.

2.1 Element Matrices

As shown in figure(2) the beam element model presented here consists of two nodes and each node has n+3 degrees of freedom. Nodal displacements are given by

$$\left\{\Delta^{(e)}\right\} = \{w_p \;\; \varphi_p \;\; u_{1p} \;\; u_{3p} \ldots u_{(2n+1)p} \;\; w_q \;\; \varphi_q \;\; u_{1q} \;\; u_{3q} \ldots u_{(2n+1)q}\}^T \tag{1}$$

where p and q are elemental nodal numbers. The axial displacement, ($u_{(2k-1)}$) of the constraining layers, the transverse displacement,(w) and the rotational angle,(φ) can be expressed in terms of nodal displacements and finite element shape functions.

$$u_{(2k-1)} = [\, N_{(2k-1)} \,] \left\{\Delta^{(e)}\right\}, \quad k=1,2 \ldots n+1,$$

$$w = [N_w] \left\{\Delta^{(e)}\right\}, \; \varphi = [N_w]' \left\{\Delta^{(e)}\right\} \tag{2}$$

where the prime denotes differentiation with respect to axial coordinate x. The shape functions are given as below.

The shape function matrices, $[N_{(2k-1)}]$ are of $1\times(2n+6)$ size with the elements $[N_{(2k-1)}]_{(1,(k+2))} = 1-\zeta$ and $[N_{(2k-1)}]_{(1,(n+k+5))} = \zeta$ respectively and all other elements are zero.

The size of the shape function matrix $[N_w]$ is $1\times(2n+6)$ with the elements $[N_w]_{(1,1)} = (1-3\zeta^2+2\zeta^3)$; $[N_w]_{(1,2)} = (\zeta-2\zeta^2+\zeta^3)\,l$; $[N_w]_{(1,(n+4))} = (3\zeta^2-2\zeta^3)$; $[N_w]_{(1,(n+5))} = (-\zeta^2+\zeta^3)\,l$ and all other elements are zero.

where $\zeta = x / l$, l is the length of the element.

2.1.1 Element Stiffness Matrix

Elemental potential energy ($U^{(e)}$) is equal to the sum of the potential energy of the constraining layers and viscoelastic layers.

$$U^{(e)} = U_c^{(e)} + U_v^{(e)} \tag{3}$$

Potential energy of the constraining layers

The potential energy of the constraining layers due to axial extension and bending is given as

$$U_c^{(e)} = \sum_{k=1}^{n+1} \frac{1}{2} \int_0^l E_{(2k-1)}\, I_{(2k-1)} \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + \frac{1}{2} \int_0^l E_{(2k-1)}\, A_{(2k-1)} \left(\frac{\partial u_{(2k-1)}}{\partial x}\right)^2 dx \tag{4}$$

where $E_{(2k-1)}$, $A_{(2k-1)} = b\{t_{(2k-1)}\}$ and $I_{(2k-1)} = b\{t_{(2k-1)}\}^3 / 12$ are the Young's modulus, cross-sectional area and area moment of inertia of the ($2k-1$)th constraining layer respectively.

Potential energy of the viscoelastic layers

The potential energy of the viscoelastic layers due to shear deformation is given as

$$U_v^{(e)} = \sum_{j=1}^{n} \frac{1}{2} \int_0^l G_{v(2j)}\, A_{v(2j)}\, \gamma_{v(2j)}^2 \, dx \tag{5}$$

where $A_{v(2j)}$ is the cross-sectional area and $G_{v(2j)}$ is the complex shear modulus of 2jth layer. $G_{v(2j)} = G^*_{v(2j)} \left[1 + i(\eta_c)_{(2j)}\right]$, where $G^*_{v(2j)}$ is the in-phase shear modulus of the 2jth viscoelastic material layer, $(\eta_c)_{(2j)}$ is the associated core loss factor and $i = \sqrt{-1}$.

The shear strain $\gamma_{v(2j)}$ of the 2jth viscoelastic layer from kinematic relationships between the constraining layers[4] is expressed as follows:

$$\gamma_{v(2j)} = \frac{u_{2j+1} - u_{2j-1}}{t_{v(2j)}} + \frac{(t_{2j+1} + 2t_{v(2j)} + t_{2j-1})}{2t_{v(2j)}} \frac{\partial w}{\partial x} \tag{6}$$

Substituting equation (2) in to equation (6) $\gamma_{v(2j)}$ can be expressed in terms of nodal displacements and element shape functions:

$$\gamma_{v(2j)} = \left[N_{\gamma(2j)}\right] \left\{\Delta^{(e)}\right\} \tag{7}$$

where $$\left[N_{\gamma(2j)}\right] = \frac{\left(\left[N_{2j+1}\right]-\left[N_{2j-1}\right]\right)}{t_{v(2j)}} + \frac{(t_{2j+1}+2t_{v(2j)}+t_{2j-1})}{2t_{v(2j)}}\left[N_w\right]' \tag{8}$$

Substituting equation (7) in to equation (5), the potential energy of the viscoelastic material layers is given by

$$U_v^{(e)} = \sum_{j=1}^{n}\frac{1}{2}\left\{\Delta^{(e)}\right\}^T \left(\left[K_{v\gamma(2j)}^{(e)}\right]\right)\left\{\Delta^{(e)}\right\} \tag{9}$$

From equation (3) elemental potential energy

$$\begin{aligned} U^{(e)} &= \sum_{k=1}^{n+1}\frac{1}{2}\left\{\Delta^{(e)}\right\}^T \left(\left[K_{(2k-1)u}^{(e)}\right]+\left[K_{(2k-1)w}^{(e)}\right]\right)\left\{\Delta^{(e)}\right\} + \sum_{j=1}^{n}\frac{1}{2}\left\{\Delta^{(e)}\right\}^T\left(\left[K_{v\gamma(2j)}^{(e)}\right]\right)\left\{\Delta^{(e)}\right\} \\ &= \frac{1}{2}\left\{\Delta^{(e)}\right\}^T\left(\left[K^{(e)}\right]\right)\left\{\Delta^{(e)}\right\} \end{aligned} \tag{10}$$

where $$\left[K^{(e)}\right] = \sum_{k=1}^{n+1}\left(\left[K_{(2k-1)u}^{(e)}\right]+\left[K_{(2k-1)w}^{(e)}\right]\right)+\sum_{j=1}^{n}\left(\left[K_{v\gamma(2j)}^{(e)}\right]\right) \tag{11}$$

$$\left.\begin{aligned} \left[K_{(2k-1)u}^{(e)}\right] &= E_{(2k-1)}\,A_{(2k-1)}\int_0^l \left[N_{(2k-1)}\right]'^T\left[N_{(2k-1)}\right]'\,dx \\ \left[K_{(2k-1)w}^{(e)}\right] &= E_{(2k-1)}\,I_{(2k-1)}\int_0^l \left[N_w\right]''^T\left[N_w\right]''\,dx \end{aligned}\right\} \tag{12}$$

$$\left[K_{v\gamma(2j)}^{(e)}\right] = \sum_{j=1}^{n} G_{v(2j)}\,A_{v(2j)}\int_0^l \left[N_{\gamma(2j)}\right]^T\left[N_{\gamma(2j)}\right]dx \tag{13}$$

$\left[K^{(e)}\right]$ is the element stiffness matrix

2.1.2 Element Mass Matrix

Elemental kinetic energy ($T^{(e)}$) is equal to the sum of the kinetic energy of the constraining layers and viscoelastic layers.

$$T^{(e)} = T_c^{(e)} + T_v^{(e)} \tag{14}$$

(i) Kinetic energy of the constraining layers is written as

$$T_c^{(e)} = \sum_{k=1}^{n+1}\frac{1}{2}\int_0^l \rho_{(2k-1)}\,A_{(2k-1)}\left(\frac{\partial w}{\partial t}\right)^2 dx + \frac{1}{2}\int_0^l \rho_{(2k-1)}\,A_{(2k-1)}\left(\frac{\partial u_{(2k-1)}}{\partial t}\right)^2 dx \tag{15}$$

where $\rho_{(2k-1)}$ is the mass density of the $(2k-1)$th constraining layer.

(ii) Kinetic energy of the viscoelastic layers is written as

$$T_v^{(e)} = \sum_{j=1}^{n}\frac{1}{2}\int_0^l \rho_{v(2j)}\,A_{v(2j)}\left\{\left(\frac{\partial w}{\partial t}\right)^2 + \left(\frac{\partial u_{v(2j)}}{\partial t}\right)^2\right\}dx \tag{16}$$

where $A_{v(2j)}$ is the cross-sectional area and $\rho_{v(2j)}$ is the mass density of the 2jth viscoelastic layer

The axial displacement $u_{v(2j)}$ of the 2jth viscoelastic layer derived from kinematic relationships between the constraining layers[3] is expressed as follows:

$$u_{v(2j)} = \frac{u_{2j+1}+u_{2j-1}}{2} + \frac{(t_{2j+1}-t_{2j-1})}{4}\frac{\partial w}{\partial x} \tag{17}$$

Substituting equation (2) in to equation (17) u_v can be expressed in terms of nodal displacements and element shape functions:

$$u_{v(2j)} = \left[N_{v(2j)}\right] \left\{\Delta^{(e)}\right\} \quad (18)$$

where $\left[N_{v(2j)}\right] = \frac{1}{2}\left(\left[N_{2j+1}\right] + \left[N_{2j-1}\right]\right)\frac{(t_{2j+1} - t_{2j-1})}{4}\left[N_w\right]'$ (19)

Substituting equation (2) in to equations (18) and (16), the kinetic energy of viscoelastic material layers is given by

$$T_v^{(e)} = \sum_{j=1}^{n} \frac{1}{2}\left\{\dot{\Delta}^{(e)}\right\}\left(\left[M_{v(2j)}^{(e)}\right]\right)\left\{\dot{\Delta}^{(e)}\right\} \quad (20)$$

From equation(14)

$$T^{(e)} = \sum_{k=1}^{n+1} \frac{1}{2}\left\{\dot{\Delta}^{(e)}\right\}^T \left(\left[M_{(2k-1)u}^{(e)}\right] + \left[M_{(2k-1)w}^{(e)}\right]\right)\left\{\dot{\Delta}^{(e)}\right\} + \sum_{j=1}^{n} \frac{1}{2}\left\{\dot{\Delta}^{(e)}\right\}^T \left(\left[M_{v(2j)}^{(e)}\right]\right)\left\{\dot{\Delta}^{(e)}\right\} \quad (21)$$

$$T^{(e)} = \frac{1}{2}\left\{\dot{\Delta}^{(e)}\right\}^T \left(\left[M^{(e)}\right]\right)\left\{\dot{\Delta}^{(e)}\right\} \quad (22)$$

where $\left[M^{(e)}\right] = \sum_{k=1}^{n+1}\left(\left[M_{(2k-1)u}^{(e)}\right] + \left[M_{(2k-1)w}^{(e)}\right]\right) + \sum_{j=1}^{n}\left(\left[M_{v(2j)}^{(e)}\right]\right)$ (23)

$$\left.\begin{aligned} \left[M_{(2k-1)u}^{(e)}\right] &= \rho_{(2k-1)} A_{(2k-1)} \int_0^l \left[N_{(2k-1)u}\right]^T \left[N_{(2k-1)u}\right] dx \\ \left[M_{(2k-1)w}^{(e)}\right] &= \rho_{(2k-1)} A_{(2k-1)} \int_0^l \left[N_w\right]^T \left[N_w\right] dx \end{aligned}\right\} \quad (24)$$

$$\left[M_{v(2j)}^{(e)}\right] = \sum_{j=1}^{n} \rho_{v(2j)} A_{v(2j)} \int_0^l \left[N_{v(2j)}\right]^T \left[N_{v(2j)}\right] dx + \rho_{v(2j)} A_{v(2j)} \int_0^l \left[N_w\right]^T \left[N_w\right] dx \quad (25)$$

and $\left[M^{(e)}\right]$ is the elemental mass matrix and the dot denotes differentiation with respect to time t.

2.2 Governing Equations of Motions

The element equation of motion for a sandwich beam is obtained by using Hamilton's principle.

$$\delta \int_{t_1}^{t_2} \left(T^{(e)} - U^{(e)}\right) dt = 0 \quad (26)$$

Substituting equations (13), (26) and (29) in to equation (30) the equation of motion for the sandwich beam element is obtained as follows:

$$\left[M^{(e)}\right]\left\{\ddot{\Delta}^{(e)}\right\} + \left[K^{(e)}\right]\left\{\Delta^{(e)}\right\} = 0 \quad (27)$$

Assembling mass, elastic stiffness and geometric stiffness matrices of individual element, the equation of motion for the beam is written as

$$[M]\left\{\ddot{\Delta}\right\} + [K]\{\Delta\} = 0 \quad (28)$$

where $\{\Delta\}$ is the global displacement matrix.

4. Results and Discussion

With a ten element discretisation of the beam, the resonant frequencies and modal system loss factors obtained for a three-layer beam are compared with those of Rao [4] and results are found to be in good agreement.

Figures (3-4) show the effect of core thickness parameter (t_{21}) on first and second modal frequency parameters η_1 and η_2 of the sandwich beam with three, five and seven layers. For all the three types of beam shear parameter $(g)_N$ has been taken as 5.0. It is observed from the graphs that for a beam of particular number of layers, both the frequency parameters increase linearly with increase in core thickness parameter. Also with increase in number of layers when core thickness parameter is same for all types of beam, both the frequency parameters increase. The increase in resonant frequencies with number of layers is more for higher values of core thickness parameter.

5. Conclusion

The developed finite element model for multilayered sandwich beam with viscoelastic core gives accurate results for frequency calculation. The first and second resonant frequencies increase with increase in core thickness parameter and number of layers. The proposed element can be used for vibration analysis of sandwich beams having any number of layers.

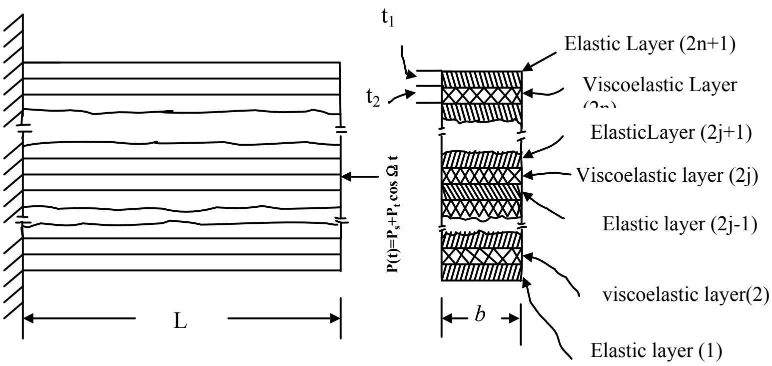

Figure - 1, Configuration of a 2n+1 layered cantilever sandwich beam

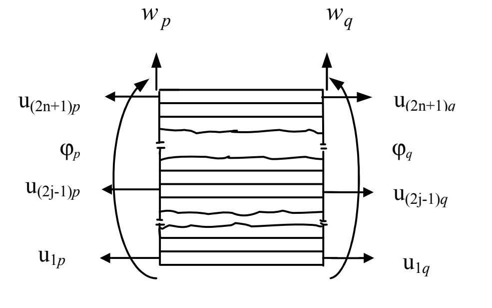

Figure - 2, Finite beam element for a (2n+1) layered sandwich beam

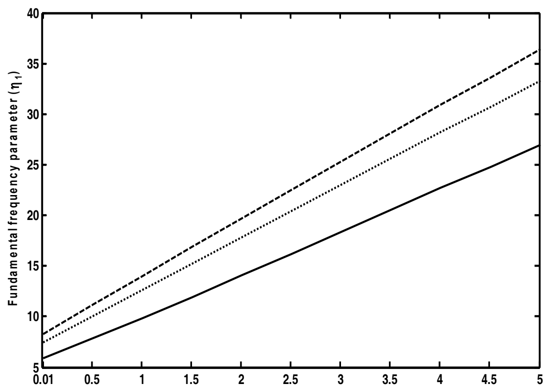

Fig.- 3,Effect of core thickness parameter on fundamental frequency parameter, $(g)_N$= 5.0,η_c= 0.18;N=3;- ,N=5; ... ,N=7;- -,.

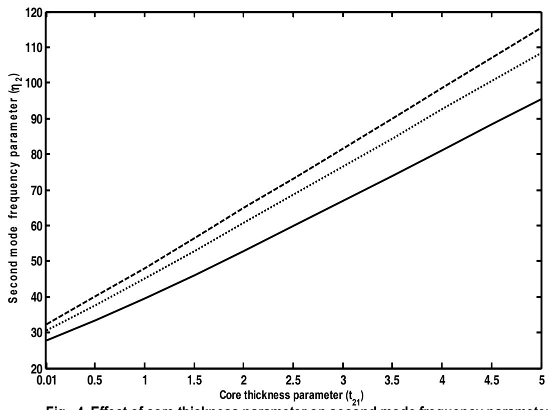

Fig.- 4 ,Effect of core thickness parameter on second mode frequency parameter $(g)_N$= 5.0, η_c = 0.18,key as fig.- 3.

6. References

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