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Allocation Marks Model for Examination Based on Bloom's Taxonomy

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Abstract. Examination is a set of questions used to evaluate skills or knowledge of students and very important for educators to determine the quality of learning process. Appropriate questions can be developed to assess the required complexity levels. The objective of this study is to allocate the optimal marks for each chapter of Engineering Mathematics II course in examination questions in order to satisfy the requirements of Bloom's Taxonomy. This paper presents two models that can assist mathematics lecturers to create optimal structure of examination marks distribution. Mathematical model is developed to determine and allocate the appropriate marks for each chapter. The results show that the optimal distributions of allocation marks are obtained for each chapter and at the same time it satisfies the requirements of percentage of complexity cognitive levels in Bloom's Taxonomy.

Keywords: Bloom's taxonomy, Examination model, Mathematical model

1. Introduction

All of Learning, teaching, identifying educational goal are important process for lecturer to consider before setting up the examination's paper. Besides, other aspects should be considered including the coverage and the depth of study for certain topic, the approach in learning particular topic and length of periods needed to finish up the topic.

One approach in determining the suitable questions for examination's paper that fulfill the above aspects is by following the system of Bloom's Taxonomy. Bloom's Taxonomy (BT) was developed by education psychologist Benjamin Bloom in 1956 to categorize intellectual skills and behavior, which are important for learning process. BT identified three domains of educational activities: cognitive, psychomotor and affective. Since, theoretical questions in examination very difficult to test for both of psychomotor and affective domain, hence, for this study, we cover on cognitive domain only. Cognitive domain involves with the development of knowledge and intellect. There are six levels in cognitive domain: knowledge, comprehension, application, analysis, synthesis and evaluation, starting from less to more complicated level.

For over 50 years, BT has strongly influence teaching and its assessment throughout the world [2] and commonly used in mathematics education. For example, Kastberg and Vidakovic [3], Bevis and Alexander [4] provide examples of how high school and collegiate mathematics teachers can use BT to develop examination or test items. Numerous studies have used BT as the standard for judging whether tested items are Lower-Order-Thinking (LOT) or Higher-Order-Thinking (HOT). The thinking skills in BT considered LOT include knowledge and understanding, while the thinking skills of analysis, synthesis and evaluation are considered HOT. Application often falls into both categories (Refer Figure 1).

The taxonomy is hierarchical means that each level is subset by the higher levels. In other words, a student functioning at the 'application' level has also mastered at the 'knowledge' and 'comprehension' levels. As originally designed, BT was an attempt to establish a sequential and cumulative hierarchy depicting the stages of learning from the most elementary to the most complex [5]. In addition, a review of

BT classification can help us, as lecturers in Institute of Engineering Mathematics, Universiti Malaysia Perlis in designing more effective questions to measure student's ability at the same time try to test them with various complexity methods in order to produce quality students.

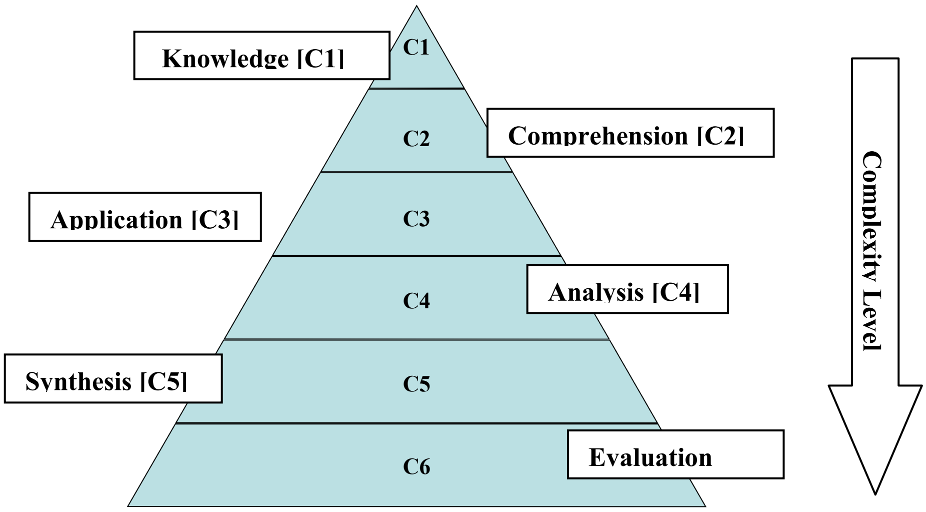

Figure 1. Six levels in cognitive domain follow to complexity hierarchy

The application of BT at a higher level is not widely use at secondary school, since at higher level every subject is under the lecturer's control. Means that, it depends on the lecturers either they want to follow BT or not in preparing exam's questions. A study done by Carolyn [6] to high school students where they are given instruction in Bloom's Cognitive Taxonomy and has been posed questions at various levels; she found that it led to richer mathematics content-related classroom discussions.

Normally to plan questions for examination, the elements that always considered to allocate mark are 1)teaching periods, 2)course outcome priority (competency and skill) and 3)difficulties. According to that the application of BT is very useful to assist lecturer to prepare the questions. Based on original purpose, Benjamin Bloom created this BT system for categorizing level of abstraction of questions that commonly occur in educational settings. The BT provides a useful structure in which to categorize test or examination questions. Since lecturer characteristically ask questions within particular levels, the lecturer will able to plan the levels of questions that will be tested in examination and at the same time the lecturer will able to study and prepare the questions using appropriate strategies. The taxonomy is presented with demonstrated skills and question cues for each level follow to Table 1.

Table 1:Bloom's Taxonomy with Levels of Abstraction of Questions Commonly Used in Education

Competence	Demonstrated Skills	Question Cues
Knowledge	• Observation and recall of information • Knowledge of dates, events, places • Knowledge of major ideas • Mastery of subject matter	*list, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, when, where*
Comprehension	• Understanding information • Grasp meaning • Translate knowledge into new context • Interpret facts, compare, contrast • Order, group, infer causes • Predict consequences	*summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend*
Application	• Use information • Use methods, concepts, theories in new	*apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change,*

	situations • Solve problems using required skills or knowledge	*classify, experiment, discover*
Analysis	• Seeing patterns • Organization of parts • Recognition of hidden meanings • Identification of components	*analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer*
Synthesis	• Use old ideas to create new ones • Generalize from given facts • Relate knowledge from several areas • Predict, draw conclusions	*combine, integrate, modify, rearrange, substitute, plan, create, design, invent, what if? , compose, formulate, prepare, generalize, rewrite*
Evaluation	• Compare and discriminate between ideas • Assess value of theories or presentation • Make choices based on reasoned argument • Verify value of evidence • Recognize subjectively	*Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize*

According to above matters, the objective of this study is to allocate the optimal marks for each chapter of Engineering Mathematics II course in examination question that satisfying the requirements of Bloom's Taxonomy. This paper presents two models that can assist mathematics lecturers to create optimal structure of examination marks distribution. Engineering Mathematics II is a compulsory course that should be taken by first year students and it is offered in every semester.

2. Model Development

In this study, two models of examination have been developed, namely Model 1 and Model 2. Both of the models ran using the application of linear programming. These models cover all chapters of Engineering Mathematics II subject; namely 1st Order Differential Equation, 2nd Order Differential Equation, Laplace Transform and Fourier Series. Since the questions makers feel that cognitive domains for analysis, synthesis and evaluation are too difficult and complex for students if being tested in the limited time examination, for examination questions, lecturers just decide to test three terms level of cognition that are knowledge, comprehension and application.

2.1. Model 1

The key decision for this model is to determine the optimal allocation marks for each question of Engineering Mathematics II course by satisfying the requirement of BT, while the allocation percentage constraints for each subtopic are determined by the questions makers.

The objective function (1) is to maximize the allocation marks for application cognitive level. The roles of the constraints (3), (4), (9), (10) and (11) are to ensure that the allocation marks for the questions of linear equation, exact equation, neither functions of Fourier Series, inverse Laplace and odd/even function of Fourier Series are greater than or equal to 10% of the total marks. Constraint (5) show that the allocation marks for 2nd Order Differential Equation: Method of Undetermined Coefficient is greater than or equal to 20%. Allocation marks for Properties of Laplace subtopic is greater than or equal to 8% as stated by constraint (6). Constraint (7) assures that the allocation marks for Application of Laplace Transform are greater than or equal to 12% the total marks. Meanwhile constraint (8) ensures that the allocation mark for properties of even and odd functions of Fourier series must greater than 5% of the total marks. Constraints (12), (13) and (14) are referred as level of cognition constraints. The allocation marks for knowledge, understanding and application level of cognition must be greater than or equal to 10%, greater than or equal to 20 % and greater than or equal to 60% of the total marks, respectively. There is no suitable question for knowledge cognitive level type for Neither Functions of Fourier Series. The combination of three cognitive levels must not exceed 20% for each question. Constraint (17) assures that the variable $x_{i,j}$ are integer.

Table 2: Model 1

			Main Variables	Allocation Percentage	Knowledge	Comprehension	Application
Main Variables					z_1	z_2	z_3
Allocation Percentage					$z_1 \geq 10\%$	$z_2 \geq 10\%$	$z_3 \geq 60\%$
1	1st order Linear Differential Equation	Linear Equation	x_1	$x_1 \geq 10\%$	x_{11}	x_{12}	x_{13}
		Exact Equation	x_2	$x_2 \geq 10\%$	x_{21}	x_{22}	x_{23}
2	2nd Order Differential Equation: Method of Undetermined Coefficient		x_3	$x_3 \geq 20\%$	x_{31}	x_{32}	x_{33}
3	Laplace Transform	Properties of Laplace	x_4	$x_4 \geq 8\%$	x_{41}	x_{42}	x_{43}
		Application of Laplace Transform	x_5	$x_5 \geq 12\%$	x_{51}	x_{52}	x_{53}
4	Fourier Series	Properties of Even and Odd Functions	x_6	$x_6 \geq 5\%$	x_{61}	x_{62}	x_{63}
		Neither function	x_7	$x_7 \geq 10\%$	x_{71}	x_{72}	x_{73}
5,6	Laplace Transform	Inverse Laplace	x_8	$x_8 \geq 10\%$	x_{81}	x_{82}	x_{83}
	Fourier Series	Odd/Even Function	x_9	$x_9 \geq 10\%$	x_{91}	x_{92}	x_{93}

Model 1 can be written as below:

Objective function

$$\text{Maximize } \sum_{i=1}^{9} x_{i3} \tag{1}$$

Subject to

Main variables constraints

$$x_i = \sum_{j=1}^{3} x_{ij} \qquad (i = 1, \ldots, 9) \tag{2}$$

Subtopics constraints

$$x_1 \geq 0.1 \sum_{i=1}^{9} x_i \tag{3}$$

$$x_2 \geq 0.1 \sum_{i=1}^{9} x_i \tag{4}$$

$$x_3 \geq 0.2 \sum_{i=1}^{9} x_i \tag{5}$$

$$x_4 \geq 0.08 \sum_{i=1}^{9} x_i \tag{6}$$

$$x_5 \geq 0.12 \sum_{i=1}^{9} x_i \tag{7}$$

$$x_6 \geq 0.05 \sum_{i=1}^{9} x_i \tag{8}$$

$$x_7 \geq 0.1 \sum_{i=1}^{9} x_i \tag{9}$$

$$x_8 \geq 0.1\sum_{i=1}^{9} x_i \quad (10)$$

$$x_9 \geq 0.1\sum_{i=1}^{9} x_i \quad (11)$$

Level of cognition constraints

$$\sum_{i=1}^{9} z_1 \geq 0.1\sum_{i=1}^{9} x_i \quad (12)$$

$$\sum_{i=1}^{9} z_2 \geq 0.2\sum_{i=1}^{9} x_i \quad (13)$$

$$\sum_{i=1}^{9} z_3 \geq 0.6\sum_{i=1}^{9} x_i \quad (14)$$

Additional constraints

$$x_{71} = 0 \quad (15)$$

$$x_i \leq 20 \qquad (i = 1,...,9) \quad (16)$$

$$x_{ij} = \text{integer} \qquad (i = 1,...,9;\ j = 1,...,3) \quad (17)$$

2.2. Model 2

The key decision for this model is to determine which subtopic should be tested so that it satisfying the requirement of BT, where the marks constraint for each subtopics are given.

The objective function (18) is to maximize the allocation marks for application cognitive level. The constraints for this model are similar to Model 1. Constraint (20) ensures that each question have only one level of cognition. Constraints (21), (22) and (23) allocate the percentage for the question of each level of cognition based on the constraints given in table 3. Meanwhile the allocation marks for each subtopic is given in table 3. Constraint (24) ensure that the allocation marks for comprehension cognitive level must bigger than knowledge cognitive level. Besides, we have restricted that properties of even and odd functions in Fourier series must be in application level of cognition.

Model 2 can be written as follow:

Objective function

$$\text{Maximize} \sum_{j=1}^{3} x_{9j} \quad (18)$$

Subject to

Main variables constraints

$$z_j = 10x_{1j} + 10x_{2j} + 20x_{3j} + 10x_{4j} + 10x_{5j} + 5x_{6j} + 15x_{7j} + 10x_{8j} + 10x_{9j} \qquad (j = 1,...,3) \quad (19)$$

Subtopics constraints

$$\sum_{j=1}^{3} x_{ij} = 1 \qquad (i = 1,...,9) \quad (20)$$

Level of cognition constraints

$$\sum_{i=1}^{9} x_{i1} \geq 0.1\sum_{i=1}^{9} x_i \quad (21)$$

$$\sum_{i=1}^{9} x_{i2} \geq 0.2\sum_{i=1}^{9} x_i \quad (22)$$

$$\sum_{i=1}^{9} x_{i3} \geq 0.6\sum_{i=1}^{9} x_i \quad (23)$$

Additional constraints

$$z_2 \geq z_1 \quad (24)$$

$$x_{63} = 1 \quad (25)$$

$$x_{ij} = \text{binary} \ (i = 1,...,9; j = 1,...,3) \quad (26)$$

Table 3: Model 2

			Main Variables	Allocation Percentage	Knowledge	Comprehension	Application
Main Variables					z_1	z_2	z_3
Allocation Percentage					$z_1 \geq 10\%$	$z_2 \geq 10\%$	$z_3 \geq 60\%$
1	1st order Linear Differential Equation	Linear Equation	x_1	10	x_{11}	x_{12}	x_{13}
		Exact Equation	x_2	10	x_{21}	x_{22}	x_{23}
2	2nd Order Differential Equation: Method of Undetermined Coefficient		x_3	20	x_{31}	x_{32}	x_{33}
3	Laplace Transform	Properties of Laplace	x_4	8	x_{41}	x_{42}	x_{43}
		Application of Laplace Transform	x_5	12	x_{51}	x_{52}	x_{53}
4	Fourier Series	Properties of Even and Odd Functions	x_6	5	x_{61}	x_{62}	x_{63}
		Neither function	x_7	15	x_{71}	x_{72}	x_{73}
5,6	Laplace Transform	Inverse Laplace	x_8	10	x_{81}	x_{82}	x_{83}
	Fourier Series	Odd/Even Function	x_9	10	x_{91}	x_{92}	x_{93}

3. Results and Discussion

The results for both models are produced by using LINGO 11.0 packages. The results are summarized in Table 4 and Table 5, respectively. Results of Model 1 and Model 2 show that all of the constraints for each subtopic and constraints for each cognitive level are fulfilled and satisfied the requirements that decided by questions makers.

Table 4: Result of Model 1

			Knowledge	Comprehension	Application
Allocation Percentage			**10**	**20**	**50**
1	1st order Linear Differential Equation	Linear Equation	10	0	0
		Exact Equation	0	10	0
2	2nd Order Differential Equation: Method of Undetermined Coefficient		0	10	10
3	Laplace Transform	Properties of Laplace	0	0	8
		Application of Laplace Transform	0	0	12
4	Fourier Series	Properties of Even and Odd Functions	0	0	5
		Neither function	0	0	15
5,6	Laplace Transform	Inverse Laplace	0	0	10
	Fourier Series	Odd/Even Function	0	0	10

Table 5: Result of Model 2

<table>
<tr><th colspan="3"></th><th>Allocation Percentage</th><th>Knowledge</th><th>Comprehension</th><th>Application</th></tr>
<tr><td rowspan="2">1</td><td rowspan="2">1st order Linear Differential Equation</td><td>Linear Equation</td><td>10</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>Exact Equation</td><td>10</td><td>0</td><td>1</td><td>0</td></tr>
<tr><td>2</td><td colspan="2">2nd Order Differential Equation: Method of Undetermined Coefficient</td><td>20</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td rowspan="2">3</td><td rowspan="2">Laplace Transform</td><td>Properties of Laplace</td><td>8</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>Application of Laplace Transform</td><td>12</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td rowspan="2">4</td><td rowspan="2">Fourier Series</td><td>Properties of Even and Odd Functions</td><td>5</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>Neither function</td><td>15</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td rowspan="2">5,6</td><td>Laplace Transform</td><td>Inverse Laplace</td><td>10</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>Fourier Series</td><td>Odd/Even Function</td><td>10</td><td>0</td><td>0</td><td>1</td></tr>
</table>

Meanwhile, from the Table 5, it shows which subtopic should be tested satisfying the both requirements of the allocation marks and BT cognition levels which have been decided by the question makers. The result of Model 2 is summarized in binary form.

4. Conclusion and Recommendations

The results for model 1 and model 2 show that the allocation marks have satisfied the requirements of percentage which have been decided by the question makers according to Bloom's Taxonomy cognitive level. A lingo 11.0 package was used to generate the data and it gives the optimal results for both models.

The results obtained from both models could assist lecturers of Engineering Mathematics II in preparing exam's questions. These two models can be used to other courses by adjusting the parameters, variables and constraints. In the future both of provided models can be used in order to plan the allocation marks for test or examination questions easily and avoid the longer time consuming.

Our team suggests that students should be tested with different types of questions according to Bloom's Taxonomy cognitive levels.

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