

# A Multi-Product Reverse Logistics Model for Third Party Logistics

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**Abstract.** Recently, according to an exhaustion of resources and environment problems, reverse logistics has been an important issue among industries in the past few years. The objective of this research is to propose a mathematical model for reverse logistics of third party logistics (3PLs). The model is developed considering uncertainty conditions with multi-product in a multi-echelon network to minimize the total cost of the reverse logistics. The developed model is solved using LINGO and the results are verified using simulation model PROMODEL. Results from the numerical analysis and simulation show that the process of reverse logistics managed by a 3PL yields better results, with total cost decreases of 13.8%.

**Keywords:** mathematical model, simulation, reverse logistics, third party logistics.

## 1. Introduction

Reverse logistics has been an important issue among industries in the past few years, thus it is viewed in the past as a secondary operation to the new product side of business, has now moved to the forefront as a strategic and integral part of the drive to increase customer satisfaction. Such progression has stretched the responsibility of producers beyond the production and distribution to the responsibility of the end of product life cycle.

Reverse logistics activities involve product returns, source reduction, conservation, recycling, substitution, reuse, disposal, refurbishment, repair, and remanufacturing [1]. Reverse logistics focuses not only on end-of-life (EOL) product but also on servicing returned products which include non-used product as one of the service provided by manufactures. Therefore, with the complexity of reverse logistics, many companies with limited resources outsource their reverse logistics operations to third-party logistics providers (3PLs). 3PLs can provide multiple logistics services including transportation, warehousing, cross-docking, inventory management, packaging, and freight forwarding services. Preferably, these services are managed or integrated by the providers.

The management of a reverse logistics network is not similar to a logistic network. Product recovery not only reverses the product stream with the consequence of recycled or non-used products, but that the process is severely complicated by high uncertainty in many factors [2]. Because of the complexity of reverse logistics, developing a mathematical model has been commonly used in order to solve the problem [1-8]. Most of the previous researches focus on developing discrete mathematical models without considering the uncertainty factors.

This research develops a mathematical model of reverse logistics network which includes customers, collecting centers, processing centers, manufactures, and landfill. The objective of the model is to minimize the total operation cost, considering multi-product, multi-echelon, and demand uncertainty to be handled by 3PLs. Numerical experiments are conducted to illustrate the developed model. The experimental data are collected from Environmental Protection Bureau (EPB), Chiayi City, Taiwan and Environmental Protection

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Administration (EPA), Taiwan to calibrate the results of the developed model. The model is solved and analyzed using LINGO software and the results are verified using simulation model PROMODEL.

## 2. Model Development

This research develops a mathematical model of reverse logistics network which includes customers, collecting centers (CC), processing centers (PC), manufactures, and landfill. The model is developed into a generic model that represents the actual concept of product returns, source reduction, conservation, recycling, substitution, reuse, disposal, refurbishment, repair, and remanufacturing in the reverse logistics.

This objective of the model is to minimize total operational cost within the network without degrading service quality for each customer and to consider different scenarios in actual processes. Operational cost considered in this study includes monthly fixed and variable costs, such as labor cost, management cost, including transportation cost and penalty cost. Monthly fixed cost in each collecting center and processing center includes labor cost and service cost, which service cost refers to cost such as electric and water bills, product process cost. These costs consider the capacity of each collecting center and processing center. Transportation cost covers cost of delivering returned products from customers to each collecting center, processing center and all the way to the landfill or to manufactures. Penalty cost presents the cost to be charged if the returned products are failed to be collected.

The mathematical model, parameters, and variables are defined as follows:

Objective function:

$$\begin{aligned} \text{Min} \quad & \sum_{j \in J} FD_j Y_j + \sum_{k \in K} FD_k Y_k + \sum_{s \in S} \pi_s \left[ \sum_{i \in I} \sum_{j \in J} \sum_{q \in Q} RD_{ijq} QA_{ijqs} + \sum_{i \in I} \sum_{k \in K} \sum_{q \in Q} RP_{ikq} QB_{ikqs} \right. \\ & + \sum_{j \in J} \sum_{k \in K} \sum_{q \in Q} DP_{jkq} QG_{jkqs} + \sum_{j \in J} \sum_{q \in Q} DL_{jq} QH_{jqqs} + \sum_{k \in K} \sum_{q \in Q} PL_{kq} QM_{kqs} \\ & \left. + \sum_{j \in J} \sum_{m \in M} \sum_{q \in Q} DM_{jmqs} QN_{jmqs} + \sum_{k \in K} \sum_{m \in M} \sum_{q \in Q} PM_{kmq} QO_{kmqs} + \sum_{q \in Q} PE_q QP_{qs} \right] \end{aligned} \quad (1)$$

Subject to:

$$\sum_i \sum_j QA_{ijqs} \leq \sum_i QZ_{iqs} \quad \forall q \in Q, s \in S \quad (2)$$

$$QP_{qs} \leq \beta_s \times \sum_i QZ_{iqs} \quad \forall q \in Q, s \in S \quad (3)$$

$$QP_{qs} + \sum_i \sum_j QA_{ijqs} + \sum_i \sum_j QB_{ijqs} = \sum_i QZ_{iqs} \quad \forall q \in Q, s \in S \quad (4)$$

$$\underline{S}_j Y_j \leq \sum_i \sum_q QA_{ijqs} \quad \forall j \in J, s \in S \quad (5)$$

$$\sum_i \sum_q QA_{ijqs} \leq \bar{S}_j Y_j \quad \forall j \in J, s \in S \quad (6)$$

$$\underline{U}_k Y_k \leq \sum_i \sum_q QB_{ikqs} + \sum_j \sum_q QG_{jkqs} \quad \forall k \in K, s \in S \quad (7)$$

$$\sum_i \sum_q QB_{ikqs} + \sum_j \sum_q QG_{jkqs} \leq \bar{U}_k Y_k \quad \forall k \in K, s \in S \quad (8)$$

$$\sum_j QH_{jqqs} + \sum_k QM_{kqs} \leq \alpha_s \times \sum_i QZ_{iqs} \quad \forall q \in Q, s \in S \quad (9)$$

$$\sum_j \sum_k QG_{jkqs} \geq \gamma_s \times \sum_i \sum_j QA_{ijqs} \quad \forall q \in Q, s \in S \quad (10)$$

$$\sum_m \sum_q QN_{jmqs} + \sum_k \sum_q QG_{jkqs} + \sum_q QH_{jqqs} \leq \sum_i \sum_q QA_{ijqs} \quad \forall j \in J, s \in S \quad (11)$$

$$\sum_m \sum_q QO_{kmqs} + \sum_q QM_{kqs} \leq \sum_i \sum_q QB_{ikqs} + \sum_j \sum_q QG_{jkqs} \quad \forall k \in K, s \in S \quad (12)$$

$$\sum_j Y_j \leq J, \sum_k Y_k \leq K \quad (13)$$

$$QA_{ijqs}, QB_{ikqs}, QG_{jkqs}, QH_{jqqs}, QM_{kqs}, QN_{jmqs}, QO_{kmqs}, QP_{qs} \geq 0 \quad (14)$$

$$Y_j, Y_k \in \{0,1\} \quad (15)$$

Parameters:

$$FD_j = \text{Fixed cost of CC } j,$$

- $FP_k$  = Fixed cost of PC k,  
 $RD_{ijq}$  = Transportation cost of product q from customer i to CC j,  
 $RP_{ikq}$  = Transportation cost of product q from customer i to PC k,  
 $DP_{jkq}$  = Transportation cost of product q from CC j to PC k,  
 $DL_{jq}$  = Transportation cost of product q from CC j to the landfill,  
 $PL_{kq}$  = Transportation cost of product q from PC site k to the landfill,  
 $DM_{jmq}$  = Transportation cost of product q from CC j to the manufacture m,  
 $PM_{kmq}$  = Transportation cost of product q from PC site k to the manufacture m,  
 $PE_q$  = Penalty cost for product q not collected,  
 $\bar{S}_j$  = Maximum capacity of CC site j,  
 $\underline{S}_j$  = Minimum capacity of CC site j,  
 $\bar{U}_k$  = Maximum capacity of PC site k,  
 $\underline{U}_k$  = Minimum capacity of PC site k,  
 $QZ_{iqs}$  = Returned product q in customer i in scenario s,  
 $\beta_s$  = Maximum percentage of product failed to be collected in scenario s,  
 $\alpha_s$  = Maximum percentage of waste from PC and CC to landfill in scenario s,  
 $\gamma_s$  = Minimum percentage of product from CC to PC in scenario s,  
 $\pi_s$  = Probability in scenario s

Variables:

- $Y_j$  = 1 if site j is open, otherwise =0,  
 $Y_k$  = 1 if site k is open, otherwise =0,  
 $QA_{ijqs}$  = Quantity of product q from customer i transferred to CC j in scenario s,  
 $QB_{ikqs}$  = Quantity of product q from customer i transferred to PC k in scenario s,  
 $QG_{jkqs}$  = Quantity of product q from CC j transferred to PC k in scenario s,  
 $QH_{jqqs}$  = Quantity of product q from CC j transferred to landfill in scenario s,  
 $QM_{kqs}$  = Quantity of product q from PC k transferred to landfill in scenario s,  
 $QN_{jmqs}$  = Quantity of product q from CC j transferred to manufacture m in scenario s,  
 $QO_{kmqs}$  = Quantity of product q from PC k transferred to manufacture m in scenario s,  
 $QP_{qs}$  = Quantity of product q failed to be transferred in scenario s.

Equation (1) is the objective function of this model, and the objective is to minimize total cost in the reverse logistics network, such as management cost in each CC and PC, transportation cost and penalty cost. Equation (2) states the balance flow from each customer to CC. Equation (3) presents the limitation of product which are failed to be collected. Equation (4) presents the constraint for total balance flow of product collected. Equation (5) and (6) are the maximum and minimum capacity constraints of CC. Equation (7) and (8) are the maximum and minimum capacity constraints of PC. Constraints of unwanted products are presented by Equation (9) and (10). Equation (11) states balance flow of products in CC. Equation (12) states balance flow of products in PC. Constraint for maximum number of CC and PC is in Equation (13). The Equation (14) and (15) are non-negative and the binary constraints.

### 3. Numerical Experiments

#### 3.1. Numerical results

The data for the numerical experiments are collected from EPB, Chiayi City, Taiwan and EPA, Taiwan. Table 1 shows the average quantity of returned products for each month in Chiayi, Taiwan. Other data including operational cost, transportation cost, and penalty cost are collected or calibrated from the known information.

Table 1 Average monthly quantity of returned products in Chiayi, Taiwan

Type of Product	Quantity (Kg)
Paper	1086321
Glass	1128105
Plastic	612936
Metal	735449
Electronic	82833

Three scenarios are designed to describe different characteristics of uncertain quantity of returned products collected from customers. The three scenarios are: (1) Scenario 1: Total products from customer are assumed in the normal condition with the probability of 50%. (2) Scenario 2: Total products from customer are assumed decreased with the probability of 30%. (3) Scenario 3: Total products from customer are assumed increased with the probability of 20%.

To analyze the current situation of reverse logistics in Chiayi City and to compare that with the reverse logistics handled by 3PLs, related conditions are assumed to represent the characteristics in each situation.

1. Condition I: Considers the current situation of general reverse logistics network in Chiayi City, which the collecting center is controlled by the government and the processing centers and manufactures are handled by private companies.
2. Condition II: Considers the entire network is managed by a 3PL.

Table 2 shows the numerical results. The processing centers required are 4, 3, and 3 for Condition I without optimum, Condition I with optimum, and Condition II, respectively. In Condition I, the results without optimum represent the current situation of reverse logistics controlled by the city government, and results with optimum indicate that the city government can reduce total cost of 7.3% if the proposed model is applied to the process of reverse logistics. In Condition II, total cost has declined 13.8 % by comparing with the current condition. Since the entire network is managed by a 3PL, fixed cost, transportation cost, and total cost are the lowest compared to those of other conditions. The results show that the optimum of the reverse logistics can be obtained with centralization of the network by a 3PL.

Table 2 The numerical results

Condition	Optimum	Processing Center	Fixed Cost (NT\$)	Transporting Cost (NT\$)	Total Cost (NT\$)	Cost %	
I	Without optimum	1,2,3,5	8170000	16674480	24844480	100%	-
	Optimum	1,3,5	6360000	16673920	23033920	92.7%	-7.3%
II	Optimum	1,3,4	6010000	15405410	21415410	86.2%	-13.8%

### 3.2. Simulation results

The numerical data collected from EPB, Chiayi indicate that after all the product collected in the collecting centers; generally the product will be kept in the collecting centers for one to three days before transporting to the processing centers. The average time for the recovery process in the processing centers is one to three days, too.

The data and results of numerical experiments are used to simulate the process of the reverse logistics in simulation model PROMODEL. In verification stage, the simulation model is tested and compared with the designed network. Fig. 1 shows the verification stage for model of condition II with 3 processing centers. As it is shown in the figure, the simulation model is able to run without any computation error. It means the model has passed verification and validation stage. When the products are processed for one day in processing centers, utility of each processing center is higher than 80%; and the utility of each processing center is over 90% when the products are kept in processing centers for 2 days. It shows that 3 processing centers for 3PLs instead of 4 processor centers for current situation in Chiayi can provide more effectiveness in reverse logistics, thus the reverse logistics managed by 3PLs can obtain better results.

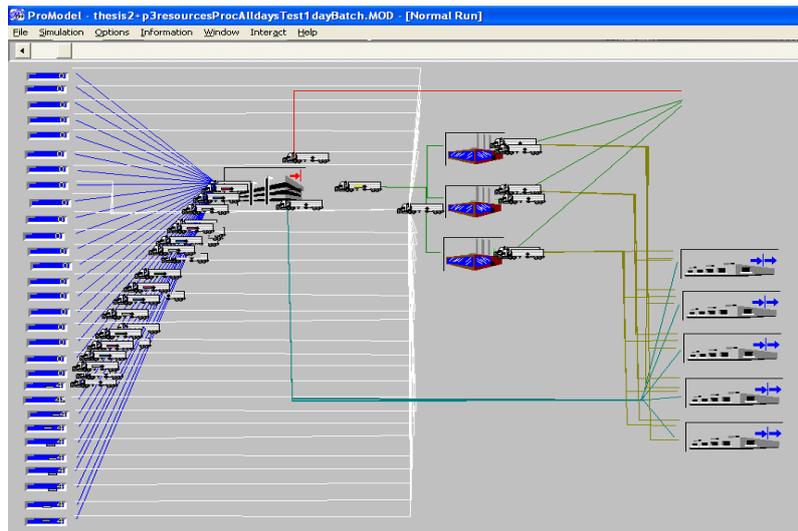


Fig. 1: Simulation process in simulation model PROMODEL

## 4. Conclusions

This research has developed a mathematical model considering uncertainty conditions with multi-product in a multi-echelon reverse logistics network. In order to validate the capability of the proposed model, numerical data collected from EPB, Chiayi City and EPA, Taiwan are applied to solve the model using LINGO software and the results are verified using simulation model PROMODEL. The results from the numerical analysis and simulation results indicate that the proposed model is able to represent a general process of reverse logistics and a practical operation of reverse logistics network managed by 3PLs. It shows that the total cost of reverse logistics can be minimized by applying the proposed model and the situation of reverse logistics managed by a 3PL yields better results, with total cost decreases of 13.8%.

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