Wide Range Stabilization Of A Magnetic Levitation System Using Analog Fuzzy Supervisory Phase Lead Compensated Controller

Huu Chan Thanh Nguyen¹, An Wen Shen²
Huazhong University of Science and Technology
Department of Control Science and Engineering,
Wuhan, China
chanthanhnguyen@gmail.com¹, sawyi@mail.hust.edu.cn²

Abstract. The magnetic levitation system is nonlinear and complex, so it is difficult to have a robust performance controller. To solve this problem, this paper presents an analog fuzzy supervisory phase lead compensated controller. The proposed controller is simulated by using MATLAB-SIMULINK and implemented on an experiment system by the combined a PIC 18F4550 microcontroller with digital controlled potentiometers. Result of experiment system shows the robustness of the analog fuzzy controller to be very good in a wide range stabilization of the levitation object position or system disturbance when structure of system is changed.

Keywords: Magnetic levitation system, phase lead compensation, analog fuzzy supervisory control.

1. Introduction

Magnetic levitation techniques have widely used in many engineering systems such as levitation of high speed trains, frictionless bearings, and wind tunnels. Magnetic levitation systems (MLS) are nonlinear and open loop unstable. Therefore, they are excellent apparatus for testing different control schemes.

In recent years, the digital controlled potentiometers (DCP) have been researched and produced widely. The combination of DCP and microcontroller gives us a flexible and cheap solution to control an analog system without converting to discrete time and using expensive digital signal processing IC.

A novel and simple design procedure based on pole cancellation method with a phase lead compensated controller [2] is proposed in this paper. The controller is easily tuned even though the system parameters are not known a priori. The numerous works [1,3] on the subject have all focused on discrete-time system analysis and design, however this paper focuses on continuous time system analysis and design an analog fuzzy supervisor phase lead compensated controller.

The second contribution of this paper is the implementation of microcontroller PIC 18F4550 combine with DCP based design, help MLS can be operated on a stand-alone basis.

2. Experimental setup and analysis

2.1. Experimental setup

The physical arrangement of the levitator is shown in Fig. 1 and the implement details on the analog fuzzy supervisor phase lead controller are provided in Fig. 2. The levitated object is a 35.2901 gram metal ball. The frame is made of wood. Referring to Fig. 1, a paired infrared emitter and receiver are employed to sense the variation of the ball position around the equilibrium point. The corresponding output range is from 8.5 to 9.3 volts, depending on the amount of lower shadow cast on the received diode. When the top of ball is exactly at the center of diodes, the output voltage is 8.9 volts. The distance between the electromagnet and
the top of ball is 4 millimeters, and the corresponding current required to balance the gravitational force is 1.023 amperes.

![Image](image1.png)

**Fig 1. Proposed magnetic levitation system**

(a) without disturbance (b) with disturbance

![Image](image2.png)

**Fig. 2. Control system block diagram of MLS**

### 2.2. Ball dynamics

A general magnetic-force mechanical dynamical equation [3] is adopted to describe the motion of the levitated object as follows:

\[
m \frac{d^2x}{dt^2} = mg - F = mg - \frac{C i^2}{x^2}
\]  

(1)

where \( m \) is the mass of the levitated object, \( g \) is the gravitational acceleration, \( x \) is the distance between the electromagnet and the levitated ball, \( i \) is the coil current and \( C \) is the force constant:

\[
C = mg \frac{X_0^2}{I_0^2}
\]  

(2)

With \( X_0 \) is the nominal operating ball position and \( I_0 \) is nominal operating current corresponding to \( X_0 \).

The researchers [2] use Taylor series expand the force \( F \) at \((X_0, I_0)\), so that:

\[
F = C \frac{I_0^2}{X_0^2} \frac{2mg}{X} (x - X_0) + \frac{2mg}{I} (i - I_0) + \text{high\_order\_terms}
\]  

(3)

Neglect the high\_order\_terms, from (1) (2) and (3), we have the formula as follow:
\[ \frac{d^2 x}{dt^2} = \frac{2g}{X_0} (x - X_0) - \frac{2g}{I_0} (i - I_0) \]  \hspace{1cm} (4)

The position of ball is voltage feedback by IR LED:

\[ V_{\text{sensor}} = -\gamma (x - X_0) \]  \hspace{1cm} (5)

The linear factor of position sensor circuit is approximated by \(-\gamma\) (volt/m).

\[ \Rightarrow \Delta V_{\text{sensor}} = -\gamma \Delta x \]  \hspace{1cm} (6)

Relation of control voltage and current through the coil:

\[ i = \beta u + I_0 \]  \hspace{1cm} (7)

With \(\beta\) (ampere/volt) is the convert voltage to current coefficient.

\[ \Rightarrow \Delta i = \beta \Delta u \]  \hspace{1cm} (8)

**Table 1. Parameters of the experimental system**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_0)</td>
<td>Setpoint</td>
<td>4</td>
<td>mm</td>
</tr>
<tr>
<td>(m)</td>
<td>Mass of metal ball</td>
<td>35.2901</td>
<td>Gram</td>
</tr>
<tr>
<td>(R)</td>
<td>Coil resistance</td>
<td>5.2</td>
<td>(\Omega)</td>
</tr>
<tr>
<td>(L)</td>
<td>Coil inductance</td>
<td>0.02045</td>
<td>H</td>
</tr>
<tr>
<td>(I_0)</td>
<td>Corresponding current at setpoint</td>
<td>1.023</td>
<td>A</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Voltage to current efficiency</td>
<td>0.55</td>
<td>A/V</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Voltage to distance efficiency</td>
<td>24.55</td>
<td>V/m</td>
</tr>
<tr>
<td>(g)</td>
<td>Gravity acceleration</td>
<td>9.81</td>
<td>m/s²</td>
</tr>
</tbody>
</table>

Combine (4) (6) and (8):

\[ \Delta \dot{V} = \frac{2g}{X_0} \Delta V + \gamma \beta \frac{2g}{I_0} \Delta u \]  \hspace{1cm} (9)

By taking the Laplace transform of (9), one can obtain the linear system transfer function:

\[ G(s) = \frac{\Delta V(s)}{\Delta u(s)} = \frac{\eta}{s^2 - \omega_o^2} \quad \text{with} \quad \eta = \frac{2\beta \gamma g}{I_0} \quad \text{and} \quad \omega_o = \sqrt{\frac{2g}{X_0}} \]  \hspace{1cm} (10)

Clearly, the poles of the loop transfer function, \(G(s)\), are derived to be:

\[ p_1 = -p_2 = \omega_o = \sqrt{\frac{2g}{X_0}} = 70 \]  \hspace{1cm} (11)

### 2.3. Phase lead compensated controller

The system is designed to carry out the major function of stabilizing the working point of the MLS. Formula (11) shows that MLS has one stable pole, while the other is still unstable and can not be stabilized simply by changing the system gain. The simplest way to stabilize the system is to use the phase lead compensated controller to cancel the unstable pole [2]. In order to pull the root-locus into the left hand plane, a zero needs to be added to the phase lead compensated controller in the left-hand plane between the first left-hand plane pole and the origin. The necessary pole required for the phase lead compensated controller is placed deeper into the left-hand plan. This will minimize the impact of the pole of the compensated controller on the root-locus.

The transfer function of the Phase-Lead Compensated Controller is shown as:

\[ G_C(s) = K_c \frac{s + 1/T}{s + 1/((\alpha T))} \quad (0<\alpha<1) \]  \hspace{1cm} (12)

In Equation (12), \(1/T\) is selected to cancel the unstable pole of (11). As a rule of thumb, \(1/(\alpha T)\) is equal to 10 times \(1/T\). To use the system parameters in Table 1, the design values of \(\alpha\) and \(T\) are 0.1007 and 0.015.
respectively. To get \( K_C \), we need to use the Routh-Hurwitz stability test to verify the closed-loop transfer function in Fig. 2 as follows:

\[
1 + G(s)G_c(s) = 1 + \frac{\eta^2}{s^2 + \omega_0^2} K_C \frac{s + 1/T}{s + 1/(\alpha T)}
\]

(13)

Obviously, the outcome of the system is stable if \( K_C > 0 \). This implies that a ball with any weight can be controlled. In my experimental system, \( K_C \) was designed at 206.55.

Because the system model is a linear approximation of a nonlinear system, it is not a good idea to attempt pole-zero cancellation by placing the zero of the phase-lead controller at the system pole of -70. If the actual system pole is to the right of -70, the zero of phase-lead controller will not cancel the pole; the zero of phase lead controller will actually lie to the left of the system pole. In this case, stability will not be achieved. To provide some robustness in the face of system model uncertainties, the zero of phase-lead controller was designed at -66.67.

![Phase lead compensated controller](image)

The phase lead controller pole was chosen to be an order of 10 away from the phase lead controller zero and, hence, was set approximately to 666.7.

From Fig. 4a, it is evident that the phase lead controller can pull the uncompensated root-locus in the right-hand plane into the left-hand plane, which indicates that the system can be stabilized.

### 3. Analog fuzzy supervisory phase lead compensated controller

The MLS is, inherently, a nonlinear system due to the characteristics of magnetic force. The phase lead compensated controller introduced in section 2 is to stabilize the linear nominal system at the specified operation condition. If the weight of the ball or the set point changes, system will go to unstable state as Fig. 4b with \( X_0 = 5\text{mm} \) and \( I_0 = 1.445\text{A} \).

To accommodate the wide range of operation conditions [1], an analog fuzzy supervisory control method was utilized as shown in Fig. 3b. From Fig. 3b, if the value of \( R_{MCP41100} \) and \( R_{MCP41010} \) change, \( T, \alpha \) and \( K_C \) of phase lead compensated controller will change corresponding:

\[
\alpha = \frac{R_2}{R_1 + R_{MCP41100} + R_2} < 1; \quad T = (R_1 + R_{MCP41100})C; \quad K_C = 1 + \frac{R_1}{R_4 + R_{MCP41010}}
\]

The fuzzy set is defined:

\[
X_0 = \{5\text{mm}, 4.5\text{mm}, 4\text{mm}\} = \{L, M, H\}; \quad X = \{5\text{mm}, 4.5\text{mm}, 4\text{mm}\} = \{L, M, H\}
\]

\[
R_{MCP41100} = \{50K, 68K, 75K, 88K, 100K\}; \quad R_{MCP41010} = \{1K, 1.2K, 1.4K, 1.6K, 1.8K\}
\]

The fuzzy quantization are defined in Fig. 5 and linguistic labels are incorporated. The fuzzy inference rules are given as Table. 2. Using the centroid defuzification method, the \( R_{MCP41100} \) and \( R_{MCP41010} \) are:

\[
R_{MCP41100} = \frac{\mu_{50K} \cdot 50 + \mu_{68K} \cdot 68 + \mu_{75K} \cdot 75 + \mu_{88K} \cdot 88 + \mu_{100K} \cdot 100}{\mu_{50K} + \mu_{68K} + \mu_{75K} + \mu_{88K} + \mu_{100K}}; \quad R_{MCP41010} = \frac{\mu_{1K} \cdot 1 + \mu_{1.2K} \cdot 1.2 + \mu_{1.4K} \cdot 1.4 + \mu_{1.6K} \cdot 1.6 + \mu_{1.8K} \cdot 1.8}{\mu_{1K} + \mu_{1.2K} + \mu_{1.4K} + \mu_{1.6K} + \mu_{1.8K}}
\]
Fig. 4. Compensated locus of the MLS

(a) $X_0=4\text{mm}$ and $I_0=1.023\text{A}$, (b) $X_0=5\text{mm}$ and $I_0=1.445\text{A}$

Table 2. Fuzzy inference rules of $R_{\text{MCP41100}}$ and $R_{\text{MCP41010}}$

<table>
<thead>
<tr>
<th>$R_{\text{MCP41100}}$</th>
<th>$X_0$</th>
<th>$H$</th>
<th>$M$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{MCP41010}}$</td>
<td></td>
<td>$\mu_{75K, \mu_{1.4K}}$</td>
<td>$\mu_{68K, \mu_{1.6K}}$</td>
<td>$\mu_{50K, \mu_{1.8K}}$</td>
</tr>
<tr>
<td>$\mu_{88K, \mu_{1.2K}}$</td>
<td>$\mu_{75K, \mu_{1.4K}}$</td>
<td>$\mu_{68K, \mu_{1.6K}}$</td>
<td>$\mu_{50K, \mu_{1.8K}}$</td>
<td></td>
</tr>
<tr>
<td>$\mu_{100K, \mu_{1.4K}}$</td>
<td>$\mu_{88K, \mu_{1.2K}}$</td>
<td>$\mu_{75K, \mu_{1.4K}}$</td>
<td>$\mu_{50K, \mu_{1.8K}}$</td>
<td></td>
</tr>
</tbody>
</table>

Where \{$\mu_{50K, \mu_{68K}}, \mu_{75K, \mu_{68K}}, \mu_{75K, \mu_{1.4K}}, \mu_{1.2K}, \mu_{1.4K}, \mu_{1.6K}, \mu_{1.8K}$\} are the values of the membership function for the fuzzy set of $R_{\text{MCP41100}}$ and $R_{\text{MCP41010}}$. The simulation result of MLS controlled by fuzzy supervisor as Fig. 6 and the experimental data are listed in Table 3. Now, the analog fuzzy supervisory phase lead compensated controller is utilized to stabilize the system globally.

4. Conclusions

In this paper, a virtual pole cancellation method with a phase lead compensated controller is able to maintain stability in a levitated ball at a fix set point. By ignoring the higher order terms in Equation (3), this system becomes nearly linear and takes the floating distance of the ball as a reference point. Once the weight of the ball or set point changes drastically, the working point will move beyond the linear boundary, rendering the compensated controller unable to stabilize the system.

To have a robust performance controller, the main contribution of this paper is designed an analog fuzzy supervisory phase lead compensated controller which is not only adaptable to great changes in the metal ball weight but is also adaptable to movements of the reference point of the floating distance.

5. References


Fig. 5. Fuzzy quantization by triangle-shaped fuzzy numbers

(a) X and X₀, (b) R_{MCP41100} and R_{MCP41010}

Fig. 6. The simulation result