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A Non-cooperative Game Theoretic Approach for Multi-cell OFDM Power Allocation

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Abstract. In this paper, we develop a distributed adaptive power allocation based on game theory in the orthogonal frequency division multiplexing (OFDM) cellular systems. In this work, power control problem is solved based on the idea of the Nash equilibrium (NE) from non-cooperative game. In this regard, a utility function is proposed that each user seeks to select a transmit power level such that maximizes its own utility by adjusting the transmit power on each subchannel. In process of power control game, the users which have better subchannel gain use lower power for transmission; When as the outlying users, due to encountering higher path loss, have low subchannel gain and consume more power to be able to guarantee wanted bit rate. Power control game provides fair data rate lower power consumption for all users in comparison with improved Water-filling (WF) approach as simulation results show.

Keywords: OFDM, power control, non-cooperative game, NE, utility function

1. Introduction

In OFDM systems, Instead of transmitting symbols sequentially over the communication channel, the channel is split into many subchannels and the data symbols are transmitted in parallel over these sub-channels. Therefore, the impact of inter-symbol interference (ISI) decreases (i.e. the fading per sub-channel is flat). So, OFDM is a promising technology for high data rate transmission in wideband wireless systems for its ability to mitigate the effects of fading and combating against inter-symbol interference. Due to these special traits, many new modern communication systems use OFDM to utilize its excellent advantages; for example WiMax (IEEE 802.16e) [1]. Resource allocation is a fundamental issue in OFDM wireless networks due to the scarce resources and existence of fading channel. Thus the main challenge in designing OFDM network is to use network resources as efficiently as possible while providing the required Quality-of-Service (QoS) by the users. Up to now, many approaches have been presented to solve resource allocation problem such as WF algorithm which is a conventional well-known method. But these methods may have some limitations. For example WF cannot ensure “fairness” among users and can’t support different QoS requirements. A new high performance and dynamic method to solve resource allocation problem is the game theory, which is mathematical tool for modelling and analyzing the interaction of two or more decision makers. Non-cooperative game and cooperative game are two models of games in game theoretic approach. In a multi-cell OFDM cellular system, mobile users may not have any knowledge about other users’ conditions and act selfishly at routine state. Such fact has motivated to adoption of distributed non-cooperative game theory. Non-cooperative game can do adaptive power control in a reasonable extent. In [2], authors propose a new method for resource allocation in multi-cell OFDM systems as new work..

But, we use different utility function from [2] and use novel improved WF method instead of conventional WF.

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In this paper, we propose a new distributed non-cooperative game theoretic based power control for resource allocation in multi-cell OFDM systems. The introduced utility function has a new formulation, which is different from other previous works in OFDM. NE is achieved in implementation of the game. The proposed approach affords feasibility and fairness. It is also showed that the game conduct user to use lower power in contrast with improved WF method. The content of the paper is organized as follows: In Section 2, we describe the system model. In Section 3, we formulate the problem as a non-cooperative game. Numerical results are given in Section 4, followed by the conclusions in Section 5.

2. System Model

2.1. SINR and Rate in OFDM Systems

In a multi-cell OFDM system, which has N co-channel cells and L OFDM subchannels in each cell are reused among multiple cells; the ith user's Signal to Interference plus Noise Ratio (SINR) at subchannel lth ($l = 1, 2, \cdots, L$) can be expressed as [3]:

$$\gamma_i^l = \frac{h_{ii}^l . p_i^l}{\sum_{j \neq i} h_{ji}^l . p_j^l + \xi} \qquad , i = 1, 2, 3, ..., N \tag{1}$$

where p_i^l is the transmit power of the ith user at its lth subchannel and h_{ji}^l denotes channel power gain from the jth user to the base station of ith user on the lth subchannel. p_j^l is the transmit power of jth interferer on the lth subchannel. ξ is the thermal noise power for all the users and subchannels. Rate adaptation such as adaptive modulation provides each sub-channel with the ability to match the effective bit rates, according to the interference and channel conditions. Quadrature Amplitude Modulation (QAM) is a modulation method with high spectrum efficiency. Without loss of generality, we assume that the output of the different adaptive modulation constellation has unit power [4]. Given a desirable rate r_i^l of M-ary Quadrature Amplitude Modulation (MQAM), the BER of the lth sub-channel of the ith user can be approximated as a function of the received SINR γ_i^l by (2) [5].

$$BER_i^l \approx \beta_1 . e^{(-\beta_2 . \frac{\gamma_i^l}{2^{r_i^l} - 1})} \tag{2}$$

With some manipulations and setting $\beta_1 \approx 0.2$ and $\beta_2 \approx 1.5$ we have $\beta_i^l = -1.6/\ln(5 \times BER)$ that is very well known and conventional form in most of references and named SINR gap. We note that the BER is equal for all users and subchannels; therefore we can express BER_i^l of all i and l with BER and hence β_i^l with β. Selecting the appropriate value of β is very important and influenced in all simulations. For example if $BER=10^{-6}$ then $\beta=0.1311$. Thus, we can extract r_i^l equation from (2) that indicates the achievable data rate per second per Hz of ith user at the lth subchannel:

$$r_i^l = \log_2(1 + \beta . \gamma_i^l) \tag{3}$$

Our system is OFDM and each user can use all subchannels in its transreceive program, therefore total data rate of ith user on all subchannels is expressed as (4):

$$r_i^{\Sigma} = \sum_{l=1}^{L} r_i^l \tag{4}$$

3. Non-cooperative Power Allocation Game

3.1. Game Theory

Game theory is a study of how to mathematically determine the best strategy for given conditions in order to optimize the outcome. The users' interaction in a wireless network can be modelled as a game in which the users' terminals are the players in the game competing for network resources.

Triplet $G = \langle N, (S_i), (u_i) \rangle$ represent a game where N= $\{1, ..., i, ..., N\}$ is the set of players/users, S_i is the finite set of actions (strategies) available to user i and u_i is preference relation of player i that is called utility function for user i. In a non-cooperative game, each users' target is to choose its strategy in such a way to maximize its own utility, i.e.:

$$\underset{s_i \in S_i}{Max}(u_i) \quad for \quad i=1,...,N \tag{5}$$

We define related important concepts, namely, a Nash Equilibrium and utility function for such game in the following [6].

3.2. Nash equilibrium and its implementation on non-cooperative power control game scheme

A NE is a state of the game where no player prefers a different action if the current actions of the other players are fixed. On the other word, a NE is a set of strategies, $\left(s_1^{NE},...,s_N^{NE}\right)$ such that no user can unilaterally improve its own utility.

$$u_i\left(s_1^{NE},...,s_{i-1}^{NE},s_i^{NE},s_{i+1}^{NE},...,s_N^{NE}\right) \le u_i\left(s_1^{NE},...,s_{i-1}^{NE},s,s_{i+1}^{NE},...,s_N^{NE}\right) \quad for\ all\ s_i \in S_i \quad and\ i=1,..., N \tag{6}$$

At NE, no user has any incentive to change its strategy. We can look at a NE as the best action that each player can play based on the given set of actions of the other players. A NE is a stable outcome of G. Each player can't profit from changing his action, and because the players are rational, this is a "steady state". However, in developing non-cooperative power control game scheme, a power vector $\left(p_1^{NE},p_1^{NE},...,p_N^{NE}\right)$ is defined as a NE for short, if for every user i for $1 \le i \le N$ and for all p:

$$\left(p_1^{NE},...,p_{i-1}^{NE},p,p_{i+1}^{NE},...,p_N^{NE}\right) \in \Omega \qquad u_i\left(p_1^{NE},...,p_{i-1}^{NE},p_i^{NE},p_{i+1}^{NE},...,p_N^{NE}\right) \le u_i\left(p_1^{NE},...,p_{i-1}^{NE},p,p_{i+1}^{NE},...,p_N^{NE}\right) \tag{7}$$

$$\underset{p_i \in P_i}{Max}(u_i) \quad for \quad i=1,...,N \tag{8}$$

where Ω is set of strategy set of game.

3.3. Utility function and Power Control

Utility function is important and essential part in development of non-cooperative game theory. It is a quantification of a player's /user's preferences with respect to certain objects. Here, we defined users' utility function as a ratio of total throughput to total power that is expressed as:

$$u_i(\mathrm{P}) = \frac{\lambda_1.f\left(r_i^{\Sigma}\right)}{\lambda_2+\lambda_3\cdot\left(p_i^{\Sigma}\right)^{\alpha}}, p_i^l \ge 0 \tag{9}$$

where p_i^{Σ} is the power summation (total power) of ith user over all sub-channels that is expressed as:

$$p_i^{\Sigma} = \sum_{i=1}^{L} p_i^l \tag{10}$$

where P ($\mathrm{P_i}=\{p_i^1,...,p_i^L\}$) denote the power allocation vector of all users at all subchannels. f is a function of r_i^{Σ}. Here $f(r_i^{\Sigma}) = r_i^{\Sigma}$. α is a non-negative real value that we can call it gain scaling parameter which is associated with power and data rate. Selection of α depends on the application. Choosing α >1places more emphasis on power usage and choosing α < 1 places more emphasis on the QoS and frame error rate. This approach used in [7], too. However, in (9), λs($\lambda_1,\lambda_2,\lambda_3$) are constant coefficient which are called *balancing factor*. To find NE point it is necessary to develop condition (11):

$$\frac{\partial u_i}{\partial p_i^l} = 0 \tag{11}$$

With some mathematical manipulations we will have:

$$\left(\partial f\left(r_i^{\Sigma}\right)/\partial r_i^{\Sigma}\right)\cdot\left(\frac{1}{\ln 2}\cdot\frac{\beta.\gamma_i^l}{1+\beta.\gamma_i^l}\cdot\frac{p_i^{\Sigma}}{p_i^l}\right) = \alpha\, f\left(r_i^{\Sigma}\right) \tag{12}$$

Equation (12) is the NE equation of the non-cooperative game scheme of (8). All users try to choose appropriate power control strategy to maximize their own utility to earn NE. In NE point of the Power control game, no single user can improve its power level by unilateral changes in its power.

4. Simulation and Numerical Results

In this section, we demonstrate the performance of our method in power allocation using our proposed utility function. In our designation, six hexagonal cells are located far away from the centre of 7th central cell with distance of 2Km. We suppose that every cell has 500m radius and consists of a base transreceiver station (BTS) at the centre of the cell and one mobile station (MS/user) is inside the cell, as seen in Fig. 1. Uplink and downlink exist between MS and BTS. 2th user has minimum distance from his BTS and 1th user has maximum distance from his BTS among all users. We define d_i as the distance from the ith user to the corresponding BTS and as seen in Fig.1 $d_2 < d_3 < d_4 < d_5 < d_6 < d_7 < d_1$. Due to this reason, path loss is exist and can be achieved according to $g_i = 0.097/d_i^4$ formulation. Thus, total path gain level of each user will decrease by increasing the distance between BTS and MS. Hence, the nearest user to the BTS will have the best path gains. Due to using the same frequency in nonadjacent cells, co-channel interference exists among cells. The channel has one-path Rayleigh distribution model with slow flat fading. Fig. 2 shows the path gain in all subchannels. It is notable that the path gain includes both path loss and slow flat fading. Background noise ξ to be 2×10^{-15}W and total subchannels are 8. Also, SINR gap is 0.1311and α=2. In our non-cooperative game scenario, the NE of the game is attained after several iterations.

The multi-cell OFDM systems require stringent power control to maintain a desirable level signal power from each user. When we developed non-cooperative game approach, desirable level is obtained when NE is attained. Also, strength of each signal (powers of subchannels of each user) effects on harm and payoff of user in the game. Strong signals due to closer users act as co-channel interference and degrade the quality of other signals. Thus, the force of a signal is a payoff for it, but is a detriment for other user in game. The power gain level of subchannels of each user showed in Fig. 2. When NE is obtained after some iteration, the game is over and as showed in Fig. 3, the power is allocated according to inverse of summation of subchannel gain level of each user (or inverse of the distance of each user). In other word, after reaching NE, non-cooperative power control game assigns larger power to users which have low total subchannel gain and low power to users which have large total subchannel gain. We can see from our simulation, the user with lower d_i is capable to consume lower transmitting power and is capable to have better SINR and thus is capable to have better bit rate. This showes game theoretic approach can increase the abilities of far user (user with low subchannel gain level). Also, the in this power control game, when NE is satisfied, each user assigns his best or near the best subchannel for transmission. Fig. 4, shows the improved WF method power allocation.

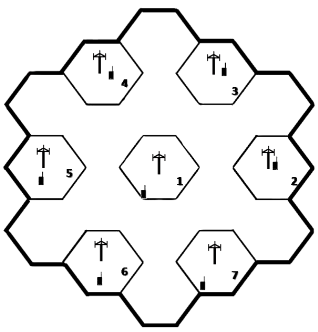

Fig. 1: Arrangement of mobile stations and BTSs

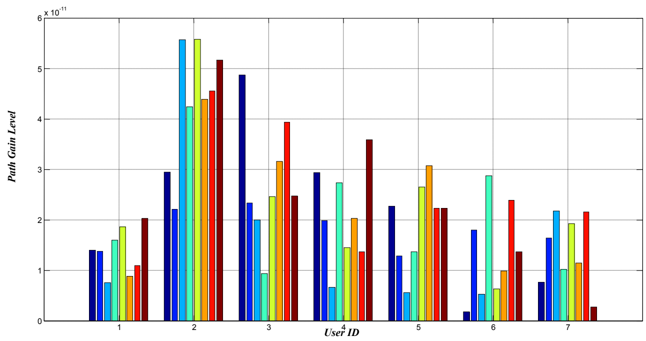

Fig. 2: Path gains of different subchannels of every user

From Fig. 4, It can be seen that, in each subchannel of every user, the allocated power level is high when users' gain is large and descend when users' gain is little; but, total power can be assigned according to the inverse manner of the subchannel, such as game method. Also, it can be seen from Fig.4, that 2th user has the most user gain and least Interference plus Noise to Signal/carrier Ratio (INSR). To comparison of WF action and non-cooperative game theoretic method, it can be seen from Fig. 5, all users have the same data rate at NE, i.e. are fair and our proposed utility based non-cooperative power control game guarantees fair data rate for all users, regardless of their distances from the BTS. It is showed that our proposed new simple utility function satisfies the power allocation demand in power allocation scenario. But WF in general use more power for each user and doesn't able to fair users' rate.

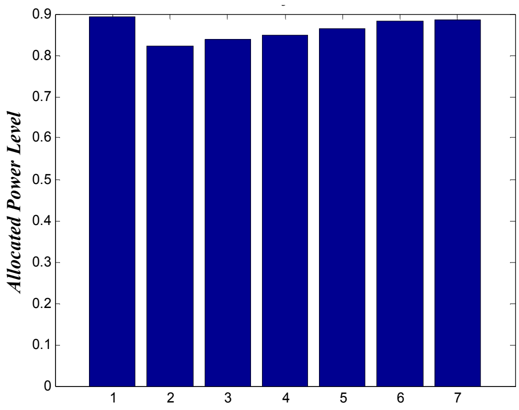

Fig. 3: allocated power in game theoretic method

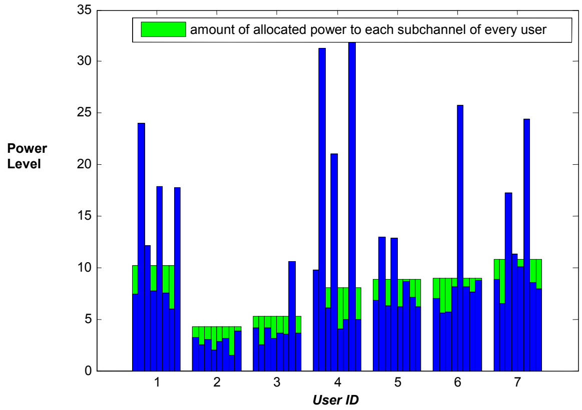

Fig. 4: WF power allocation

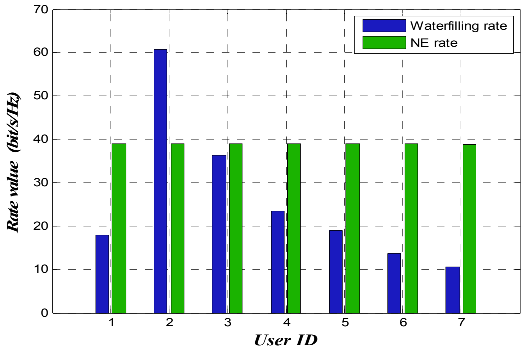

Fig. 5: Comparison between total data rate in non-cooperative power control game at NE and WF method. The NE rate is fair (equal for all users).

5. Conclusion

In this paper, we modeled a distributed non-cooperative game for power control in multi-cell OFDM systems. The NE achieved in the game. Simulations results show that the proposal achieves much better power allocation and rate in contrast with improved WF method. Game method forms fair desirable rate among users with lower power consumption.

6. References

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